EFFECT OF RANDOM STRENGTH PARAMETERS OF FLANGE STEEL ON BENDING RESISTANCE AND DEFLECTIONS OF GIRDERS WITH CORRUGATED WEB

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Received: 2.09.2020; Revised: 16.11.2020; Accepted: 18.11.2020

Abstract
The study presents results of statistical investigations into random strength parameters of steel used to manufacture flanges of plate girders with corrugated web. In the case of plate girders with corrugated webs, flanges are manufactured from flat sheet steel S235JRG2 or S355J2+N or S275. The guaranteed yield strength $R_{min}$ is determined on the basis of tests conducted on steel samples. It should be lower than yield strength of flat sheets obtained as the result of testing mechanical properties. This value at the same time should be regarded as the margin of resistance of girders. The material tests were performed on samples randomly collected from 20 girders with corrugated web and flange thickness of 15 and 20 mm, which had been already examined. Coefficients of variation of yield strength $V_{Re} = D(Re)/E(Re)$ and partial factors of yield strength $\gamma_m$ were selected on the basis of the conducted analysis of material tests. The obtained results were related to factors $\gamma_m$ determined from the tests on thin flat sheets. The impact of yield strength and random cross-sectional area of flanges on bending resistance of girders with corrugated web was shown. Also the effect of the spread of the Young’s modulus values and semi-rigid connections on global displacements of tested plate girders with corrugated web was analyzed.

Keywords: Girders with corrugated web; Normal distribution; Partial safety factor; Bending resistance.

1. INTRODUCTION

Currently manufactured girders with sinusoidal corrugated web are available with three basic thickness values of web $t = 2.0, 2.5$ and $3.0$ mm. Webs of $1.5, 4, 5$ and $6$ mm thickness can also be ordered. The thickness of girder flanges varies from $8$ to $30$ mm with the width within the range of $160–300$ mm. The height of girder web varies from $333$ to $1500$ mm at the maximum length of elements up to $20$ m.

Axial forces and bending moments in plate girders with corrugated webs are applied through flanges, and shear forces through the corrugated web. Tests on resistance and stiffness of plate girders with corrugated webs usually focus on problems concerning the loss of stability and shear resistance of the corrugated web [1–7]. In the case of bending resistance, lateral-torsional buckling of the compression flange of plate girders [8] and not thoroughly tested relationship between torsional buckling of the compression flange and corrugation of the web [9] are analysed. However, problems with the compensation for bending and shear resistance of girders are observed for plate girders with corrugated webs. There is also no reference to random parameters of steel used in flanges and their effect on bending resistance of plate girders. The possible effect on displacements particularly of long girders is an important parameter.

In the case of plate girders with corrugated webs, flanges are manufactured from flat sheet steel S235JRG2 or S355J2+N. Flanges made of steel S275 are available on request. The guaranteed yield strength of delivered hot rolled flat steel sheet known as the specified minimum $R_{min}$ equals to $235, 275$ or $355$ MPa [10]. $R_{min}$ is determined only on the basis of
tests conducted on steel specimens. Steel sheets with yield strength below the minimum value are rejected from the production. The guaranteed yield strength \( R_{\text{min}} \) should be lower than yield strength of flat sheets obtained as the result of testing mechanical properties [11, 12]. This value at the same time should be regarded as the margin of shear resistance of girders.

Flanges are connected to webs with single-sided welds at a specially prepared assembly stand (Fig. 1). So, flanges are prevented from the excessive amount of heat that can change mechanical properties of steel.

This paper describes statistical tests on random parameters of strength of sheet steel in flanges of plate girders with corrugated webs. The effect of strength parameters of flange steel on bending resistance and displacement of plate girders with corrugated web was analysed. The tests on strength parameters were performed on randomly collected samples of 20 girders with corrugated webs which had been already tested. Based on the tests, partial factors of yield strength \( \gamma_{\text{m}} \) and coefficients of variation of yield strength \( V_{\text{Re}} \) were estimated. The obtained results were related to factors determined from the tests on flat sheets [13, 14]. Also the effect of the spread of the Young’s modulus values and semi-rigid connections on global displacements of tested plate girders with corrugated web was analyzed.

2. BENDING RESISTANCE OF PLATE GIRDERS WITH CORRUGATED WEB

Bending resistance of plate girders with corrugated web is connected with resistance of flanges. The applicable design methods can be found in German guidelines DASg 015 [17], EC 3 [18] or the paper by Siokola [8]. These references include phenomena related to the impact of local or global buckling of compression flanges on bending resistance.

The method of calculating bending resistance specified in EN 1993-1-5 [18] is similar to German guidelines DASg-015 [17]. This method consists of designing tension and compression flanges of the girder taking into account local and global buckling. In the case of the compression flange, EC3 [18] associates resistance only with the reduction in yield strength considering the local buckling:

\[
N_{b,\text{Rd}} = b_{f} t_{f} f_{y} \frac{f_{T}}{\gamma_{M0}},
\]  

where: \( f_{T} \) – reduction factor for design yield strength \( f_{y} \) (equal to 1 for webs with sinusoidal wave), \( b_{f}, t_{f} \) – dimensions of the compression flange, \( \gamma_{M0} \) – partial factor equal to 1 acc. to EC [18].

In the case of lateral buckling of the compression flange concerning flexural buckling, the resistance according to EC3 [19] is expressed with the following equation:

\[
N_{u,\text{Rd}} = b_{f} t_{f} \chi \frac{f_{y}}{\gamma_{M1}},
\]

where: \( \chi \) – buckling coefficient, \( \gamma_{M1} \) – partial factor equal to 1 acc. to EC [19].

Statistical studies on strength of metallurgic products made of structural steel are described in the papers [13, 14, 15, 16]. They also include strength parameters of flat sheets. These tests are the base to determine coefficients of variation of yield strength \( V_{\text{Re}} = D(R_{e}) \) and find normal distributions of yield strength. The mean value of yield strength \( R_{e} \) is denoted as \( E(R_{e}) \), and \( D(R_{e}) \) specifies standard deviation of yield strength \( R_{e} \) (notations in accordance with quantile algebra). Coefficients of variance \( V_{\text{Re}} \) are used to calculate partial factors of yield strength \( \gamma_{m} \).
Following the purpose of Eurocode, the design bending resistance $M_{Rd}$ should be hence determined as the lowest of three values calculated from the following equation:

$$
M_{Rd} = \begin{cases} 
    b_t f_t \frac{f_T y}{\gamma_{M0}} \left( h_w + \frac{t_f + t_f}{2} \right), & \text{tension flange} \\
    b_{fc} f_{fc} \frac{f_T y}{\gamma_{M0}} \left( h_w + \frac{t_f + t_f}{2} \right), & \text{compression flange} \\
    b_{fc} f_{fc} \frac{2 f_T}{\gamma_{M1}} \left( h_w + \frac{t_f + t_f}{2} \right), & \text{compression flange} 
\end{cases}
$$

(3)

Eurocode neglects the effect of local torsional buckling on the flange resistance for calculations made for the compression flange. In the case of the global buckling, only the flexural buckling is taken into account neglecting lateral torsional buckling.

The method presented by Siokola [8] includes both the local torsional buckling of the flange, and its impact on the compression flange.

Compression resistance related to the local buckling of the compression flange is described by Siokola [8] using the following formula:

$$
N_{L,Rd} = \sigma_{cr,fc} b_{fc} f_{fc},
$$

(4)

where $b_{fc}$ and $t_{fc}$ are the width and thickness of the compression flange respectively, $\sigma_{cr,fc}$ refers to critical stresses during local buckling of the flange according to DIN 1880 Part 2 [20].

To determine the impact of global bending stability of the compression flange, Siokola applied the theory of bending in-plane buckling of bar cross-sections neglecting the commonly used truss model. Hence, the bending resistance for plate girders with the corrugated web can be determined by the following formula [8]:

$$
M_{ED} \leq M_{b,Rd},
$$

(5)

where $M_{ED}$ – moment transferred by flanges, $M_{b,Rd}$ – bending resistance of the element, including lateral-torsional buckling, that is:

$$
M_{b,Rd} = \gamma_{LT} M_{pl,y},
$$

(6)

which depends on reduction factor for lateral-torsional buckling $\gamma_{LT}$ and plastic moment of resistance of the cross-section $M_{pl,y}$ to axis $y$ without the contribution from the web which is determined from the equation:

$$
M_{pl,y} = R_{ek} \left[ b_{fc} f_{fc} \left( h_w + \frac{t_f + t_f}{2} \right) \right],
$$

(7)

where: $R_{ek}$ is characteristic yield strength of the flange, $\gamma_{M1} = 1 - \text{partial factor used to verify lateral-torsional buckling resistance}.$

It should be noted, however, that this method has been directly transferred from the design for plate girders with flat webs. As Yu and Sause described in the paper [21], this method understates results for girders with the length up to 12 m. Formulas acc. to DIN correspond to test results when girder span exceeds 12 m. It clearly indicates the effect of the support for the compression flange by the corrugated web as in the case of local torsional buckling. Similar conclusions were presented in the paper by Sherif [22] who related the obtained results to calculations made in accordance with EC3 [18].

In the case of erection load, Kowal proposed in 2005 [23] description of buckling.

According to recommendations specified in DIN 1880 Part 2 [20], there is the possibility of mutual interaction between local and global buckling mode of the compression flange of the girder. Then, the value of critical moment of lateral-torsional buckling has to be reduced to the value obtained from the equation:

$$
M_{eff,cr} = M_{cr} \sqrt{\frac{1}{1 + \left( \frac{M_{cr}}{M_{cr,L}} \right)^2}},
$$

(8)

where: $M_{cr,L}$ is critical moment inducing local torsional buckling of the compression flange:

$$
M_{cr,L} = k_\sigma \sigma_c W_{el,y}.
$$

(9)

where: $\sigma_c$ – Euler’s critical stress, $W_{el,y}$ – elastic indicator of sectional resistance, $k_\sigma = 0.6$ – parameter of slab instability.

And the ultimate limit resistance for the compression flange restrained in the lateral direction within the span $c_b$ is expressed as:
where: $c_b$ – length of lateral torsional buckling section, $k_c$ = parameter depending on distribution of normal stresses along the flange length according to DIN 1880 Part 2 \[20\].

Therefore, the resistance of corrugated web girder taking into account the tension flange, the compression flange subjected to local and global buckling, should be estimated from the following formula:

$$N_{b,Rd} = \frac{\pi}{2\sqrt{12}} \sqrt{E \cdot R_{ek}} \frac{t_{fc} b_{fc}^2}{c_b k_c}, \hspace{1cm} (10)$$

According to Siokola \[8\], the bending resistance comes down to the following formula:

$$\frac{N_{ED}}{N_{u,Rd}} < 1.$$ \hspace{1cm} (12)

where: $N_{ED}$ – maximal axial force in the flange

All quoted solutions refer to the effect of local and global buckling on the compression flange. However, they do not depreciate the positive effect of yield

![Figure 2.](image)

Corrugated web girders, from which samples were collected for testing strength parameters of flanges: a) with a semi-rigid end stiffener; b) with end stiffener reinforced by tee-bar; c) with a cantilever
strength of flanges. They only introduce the reduction factor of yield strength when local buckling of the compression flange can be expected.

3. EXPERIMENTAL TESTS ON PLATE GIRDERs WITH CORRUGATED WEB

Prior to material testing of strength parameters of steel used in flanges, tests were at first conducted on resistance of girders with the corrugated web. The effect of stiffening at the support on critical shear resistance of the corrugated web has been analyzed. Thus, three different types of corrugated web girders were selected depending on stiffening at the support (Fig. 2):  

a) girders with a semi-rigid end stiffener;  
b) girders with end stiffener reinforced by tee-bar;  
c) girders with cantilever connected with the span through the stiffener. The tests were performed on 20 girders in total. The girders were selected to ensure that failure was determined by shear resistance, and the nominal flange resistance was greater. The difference between the reached resistance considering strength parameters of steel used in flanges and the nominal bending resistance was described in Chapter 6.

The static scheme of a simply supported beam (Figs. 2a and 2b) and a simply supported beam with a single cantilever was assumed for all girders. The program of investigations is presented in Table 1.

Plate girders with the corrugated web were designed and performed in accordance with the current literature and standards [18, 24]. Girder flanges were made from hot rolled flat sheets of S275JRG2 with a thickness of 15 or 20 mm, while corrugated webs were made from hot rolled flat sheets of S235JR2G with a thickness of 2; 2.5, and 3 mm [25]. Individual plate girders were from different batches. Certificates confirming steel grade were attached to each batch. The tested group of corrugated web girders included eighteen girders with flanges of 300 x 15 mm, and two girders with flanges of 300 x 20 mm (Table 1).

Plate girders with corrugated web were assembled from three items (Fig. 2) prepared in SIN girder production plant. Individual items of tested girders were assembled with end plate connection by means of M20 or M24 high strength prestressing bolts of class 10.9. In the case of the girders with end stiffeners (Figs 2a and b), load exerted by a pair of forces 2 x P/2 (P) was transferred from the frame (FR) (Fig. 3) by means of the actuator (1) through a dynamometer (2) to the beam (3), and then to the tested girder (4) at the location of indirect stiffeners. Load in the case of cantilever girders was transferred as the force P from the frame (FR) by means of the actuator (1) through a dynamometer (2) to the endplate of the girder stiffener. The dynamometers (2) for recording reaction versus load P were located below the stiffener.

The analysed girders (Fig. 3) were tested until the ultimate resistance was reached determined by the failure of the web and formation of plastic hinges in the span or cantilever part of the flange. The samples for material testing of steel in flanges affecting the ultimate resistance of the girders, were collected from the tested girders, undamaged parts of items subjected to failure.

4. MATERIAL TESTS ON STEEL USED IN FLANGES OF SIN GIRDERs

Three flange samples were collected from each tested plate girder with the corrugated web. In the first stage, pieces of flanges were cut out from undamaged area of items of the girders (Fig. 4).
Then, the samples were cut out from flanges in accordance with the standards [24, 26]. The samples for material testing were cut out from the flanges providing their edges did not touch the plastic hinge formed in the flange. Moreover, the strength parameters of the flanges could be also affected by the zone of thermal effects of the web to flange weld. Thus, the samples were cut out near the flange edges. All samples were mechanically processed using a milling machine.

Random strength parameters of steel in flanges were tested in accordance with the standard [26]. The samples of “5-fold” base were cut out from the flanges (Fig. 5). Geometric dimensions of flange samples were measured using a caliper with a scale of 0.1 mm. In total, 60 samples were tested, including: 18 x 3 = 54 samples having a nominal thickness of 15 mm, and 2 x 3 = 6 samples with a nominal thickness of 20 mm. Results obtained from the tests on flange samples were affected only by the direction of hot rolling of steel sheet.

The tests on strength parameters of the flange samples were conducted using the test machine PUL 400 VEB Werkstoffprüfmaschinen Leipzig. An attempt was made not to exceed the stress increment rate of 8 MPa/s throughout the testing cycle. And the measurements were performed for tensile force $F$ and elongation $\Delta L (L_u - L_0)$ based on datum for the specimen $L_0$ (Fig. 6). Measurement results of mean values of yield strength $\bar{R}_e$, tensile strength $\bar{R}_m$, expansion $\bar{A}_{10}$, and Young modulus $\bar{E}$ from three samples collected from each girder are shown in Table 2.

All tested samples of steel collected from flanges of
Table 2.  
Average parameters of yield strength, tensile strength and modulus of Young tests on flange samples

<table>
<thead>
<tr>
<th>Girder No. of sample</th>
<th>$\bar{\sigma}_0$</th>
<th>$\bar{a}_0$</th>
<th>$\bar{\sigma}_0$</th>
<th>$\bar{A}_0$</th>
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Average value for samples 15 mm:

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</table>

corrugated web girders had a proper steel structure. Discards of steel samples from flanges were regular and unambiguously indicated homogeneity of the material (Fig. 6).

The tests conducted on tension of steel samples from the flanges were used to draw graphs illustrating relationships between stress and strain $\sigma - \varepsilon$ for all tested samples. Values of stress $\sigma_{l}, \sigma_{e}, \sigma_{d}$, and $\sigma_{EL}$ and $\sigma_{m}$ corresponding to yield strength and tensile strength were plotted on these graphs. It should be mentioned that the flow area separating upper and lower yield strength in the case of all tested steel samples collected from the flanges, was represented by the Lüders-Czernov curves. A difference was found between the upper and lower yield strength. Figures 7 and 8 show examples of $\sigma - \varepsilon$ graphs for tension tests on the samples and the Lüders-Czernov curves separating the upper and the lower yield strength of the samples collected from M 1.41 and M 2.42 girders. The average elongation range of tested flange steel was a bit narrower when compared to cold rolled steel used for corrugated webs [28], and varied from 24 to 33%. It meant that steel of flanges had adequate plastic properties.
5. ANALYSIS OF PARTIAL FACTORS \( \gamma_m \) AND COEFFICIENT OF VARIATIONS \( V_{Re} \) OF YIELD STRENGTH

The guaranteed yield strength of delivered flat sheets used for preparing flanges of corrugated web girders was defined by the manufacturer as the specified minimum \( R_{\text{min}} = 275 \text{ MPa} \) in accordance with the standards [10, 12].

The parameters of normal distribution were applied for the probability density distribution of yield strength \( R_y \) [12]. Variance \( D(R_y) \) and standard deviation \( D(R_y) \) were determined for yield strength of the tested samples of girder flanges. Using the parameters of normal distribution of yield strength \( R_y \), characteristic values \( R_{ek} \) (lower quantiles 5%) were determined in accordance with the standard EN 1990 [12]:

\[
R_{ek} = E(R_y)(1 - 1.64V_{Re}),
\]

where: \( E(R_y) \) – mean value of yield strength \( R_y \); \( V_{Re} \) – coefficients of variation.

Coefficients of variation of yield strength representing the safety level were determined in the first place as true \( V_{Re} \) obtained on the basis of the material tests. To compare the safety level regarding quantities used to calculate bending resistance, the coefficients of variation of yield strength were referred to the specified minimum yield strength of girders \( R_{\text{min}} = 275 \text{ MPa} \) guaranteed by the manufacturer:

\[
V_{Re} = \frac{D(R_y)}{E(R_y)} \quad \text{and} \quad V_{Re275} = \frac{1 - \frac{R_{\text{min}}}{E(R_y)}}{1.64}, \quad (14)
\]

And partial factors of yield strength \( \gamma_m \) describing the relationship between the characteristic and design yield strength were determined from the equation (15) that was obtained by transforming the standard design formula of yield strength \( f_y \) [8]:

\[
\gamma_m = \frac{R_{ek}}{f_y} = \frac{R_{ek}}{E(R_y) - 3.04D(R_y)} , \quad (15)
\]

where: \( \gamma_m \) – partial factor of yield strength \( R_y \); \( D(R_y) \) – standard deviation of yield strength \( R_y \); \( f_y \) – design yield strength.

The partial factor of yield strength \( \gamma_{m275} \) illustrating the safety level of the structure and referring to the specified minimum \( R_{\text{min}} = 275 \text{ MPa} \) based on the yield strength assumed by the manufacturer, was determined from the relationship (5) in accordance with the paper [8]:

\[
\gamma_{m275} = \frac{R_{\text{min}}}{E(R_y) - 3.04D(R_y)} , \quad (16)
\]

Figure 9 illustrates normal distributions of yield strength obtained for the samples cut out from the flanges of girders with cantilever corrugated webs and for the whole tested population of the girders. Graphs in Figure 9 present distributions of mean value \( E(R_y) \) of yield strength and design yield strength \( f_y \).

The parameters of normal distribution and determined partial yield strength factors of the flange steel samples from corrugated web girders are shown in Table 3. Mean coefficients of variation \( V_{Re} \) within the range of 0.01 < \( V_{Re} < 0.03 \), determined on the basis of the performed tests, show the typical safety margin for the structure obtained during calculations of bending resistance of the tested girders in respect of the applied yield strength. And the coefficients of variation referred to yield strength guaranteed by the manufacturer varied within the range of 0.03 < \( V_{Re} < 0.1 \). Thus, they were three times higher. The negative coefficients of variation for the girder M 1.32 was an exception that indicated the underes-
EFFECT OF RANDOM STRENGTH PARAMETERS OF FLANGE STEEL ON BENDING RESISTANCE AND DEFLECTIONS OF GIRDERS...

The estimated yield strength with reference to the guaranteed one.

Similar conclusions can be drawn by analyzing partial factors of yield strength $\gamma_m$ that are illustrated in Fig. 10. The boundary line of the partial factor of yield strength separating the characteristic value of yield strength from the design one is marked with the continuous line. Values of partial factor determined from the tests were within the range $\gamma_m = 1.01 - 1.05$ and were comparable to the factors determined from the statistical tests [13, 14, 15, 16].

The partial factors $\gamma^{275}$ referred to yield strength guaranteed by the manufacturer were in 75% smaller than 1.0, that is, smaller than the value recommended in the standards [11, 12], and the remaining 25% were greater than 1, which did not correspond to the partial factors $\gamma_m$ determined from the discussed tests. Therefore, these results led to partial overestimation of bending resistance of the tested girders.

![Figure 9. Normal distribution of yield strength $f_y$, $R_e$ from flanges SIN girders: a) from cantilever girders M.12-M 2.52; b) from all girders M.11-M 2.52](image)

Table 3. The parameters of normal distribution of yield strength $R_e$ of flanges samples from SIN girders

<table>
<thead>
<tr>
<th>Girder No. of sample</th>
<th>$E(R_e)$ [MPa]</th>
<th>$R_{ek}$ [MPa]</th>
<th>$f_y$ [MPa]</th>
<th>$D^2(R_e)$</th>
<th>$D(R_e)$</th>
<th>$V_{Re}$</th>
<th>$V_{Re^{275}}$</th>
<th>$\gamma_m$</th>
<th>$\gamma_m^{275}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 1.11</td>
<td>298.2</td>
<td>290.2</td>
<td>283.4</td>
<td>23.68</td>
<td>4.87</td>
<td>0.016</td>
<td>0.047</td>
<td>1.024</td>
<td>0.970</td>
</tr>
<tr>
<td>M 1.21</td>
<td>303.4</td>
<td>299.3</td>
<td>295.8</td>
<td>6.18</td>
<td>2.49</td>
<td>0.008</td>
<td>0.057</td>
<td>1.012</td>
<td>0.930</td>
</tr>
<tr>
<td>M 1.41</td>
<td>281.2</td>
<td>269.0</td>
<td>258.5**</td>
<td>35.78</td>
<td>7.47</td>
<td>0.027</td>
<td>0.013</td>
<td>1.040</td>
<td>1.064</td>
</tr>
<tr>
<td>M 1.51</td>
<td>291.1</td>
<td>283.0</td>
<td>276.1</td>
<td>24.52</td>
<td>4.95</td>
<td>0.017</td>
<td>0.034</td>
<td>1.025</td>
<td>0.996</td>
</tr>
<tr>
<td>M 2.11</td>
<td>313.3</td>
<td>305.3</td>
<td>295.8</td>
<td>23.74</td>
<td>4.87</td>
<td>0.016</td>
<td>0.075</td>
<td>1.023</td>
<td>0.921</td>
</tr>
<tr>
<td>M 2.21</td>
<td>322.0</td>
<td>311.4</td>
<td>302.4</td>
<td>41.43</td>
<td>6.44</td>
<td>0.020</td>
<td>0.089</td>
<td>1.030</td>
<td>0.909</td>
</tr>
<tr>
<td>M 2.31</td>
<td>328.0</td>
<td>317.0</td>
<td>307.7</td>
<td>44.76</td>
<td>6.69</td>
<td>0.020</td>
<td>0.099</td>
<td>1.030</td>
<td>0.894</td>
</tr>
<tr>
<td>M 2.41</td>
<td>326.7</td>
<td>310.7</td>
<td>297.1</td>
<td>94.99</td>
<td>9.75</td>
<td>0.030</td>
<td>0.096</td>
<td>1.046</td>
<td>0.926</td>
</tr>
<tr>
<td>M 2.51</td>
<td>290.3</td>
<td>280.5</td>
<td>272.1**</td>
<td>35.7</td>
<td>5.97</td>
<td>0.021</td>
<td>0.032</td>
<td>1.031</td>
<td>1.011</td>
</tr>
<tr>
<td>M 1.12</td>
<td>297.5</td>
<td>289.2</td>
<td>282.0</td>
<td>25.94</td>
<td>5.09</td>
<td>0.017</td>
<td>0.046</td>
<td>1.025</td>
<td>0.975</td>
</tr>
<tr>
<td>M 1.22</td>
<td>287.2</td>
<td>275.3</td>
<td>265.0**</td>
<td>53.13</td>
<td>7.29</td>
<td>0.025</td>
<td>0.026</td>
<td>1.039</td>
<td>1.038</td>
</tr>
<tr>
<td>M 1.42</td>
<td>302.8</td>
<td>287.9</td>
<td>275.1</td>
<td>82.76</td>
<td>9.10</td>
<td>0.030</td>
<td>0.056</td>
<td>1.046</td>
<td>0.999</td>
</tr>
<tr>
<td>M 1.52</td>
<td>312.5</td>
<td>301.9</td>
<td>292.9</td>
<td>41.57</td>
<td>6.45</td>
<td>0.021</td>
<td>0.073</td>
<td>1.031</td>
<td>0.939</td>
</tr>
<tr>
<td>M 2.12</td>
<td>311.2</td>
<td>302.3</td>
<td>294.7</td>
<td>29.58</td>
<td>5.44</td>
<td>0.017</td>
<td>0.071</td>
<td>1.026</td>
<td>0.933</td>
</tr>
<tr>
<td>M 2.22</td>
<td>323.5</td>
<td>310.3</td>
<td>299.0</td>
<td>65.04</td>
<td>8.06</td>
<td>0.025</td>
<td>0.091</td>
<td>1.038</td>
<td>0.920</td>
</tr>
<tr>
<td>M 2.32</td>
<td>293.9</td>
<td>282.6</td>
<td>272.9**</td>
<td>47.93</td>
<td>6.92</td>
<td>0.024</td>
<td>0.039</td>
<td>1.036</td>
<td>1.008</td>
</tr>
<tr>
<td>M 2.42</td>
<td>301.5</td>
<td>300.0</td>
<td>298.7</td>
<td>0.86</td>
<td>0.93</td>
<td>0.003</td>
<td>0.054</td>
<td>1.004</td>
<td>0.921</td>
</tr>
<tr>
<td>M 2.52</td>
<td>306.7</td>
<td>298.7</td>
<td>292.0</td>
<td>23.52</td>
<td>4.85</td>
<td>0.016</td>
<td>0.063</td>
<td>1.023</td>
<td>0.942</td>
</tr>
<tr>
<td>Average values 15 mm</td>
<td>305.1</td>
<td>295.2</td>
<td>286.9</td>
<td>40.06</td>
<td>5.98</td>
<td>0.020</td>
<td>0.059</td>
<td>1.029</td>
<td>0.961</td>
</tr>
<tr>
<td>M 1.31</td>
<td>298.9</td>
<td>287.6</td>
<td>278.03</td>
<td>47.11</td>
<td>6.86</td>
<td>0.023</td>
<td>0.049</td>
<td>1.035</td>
<td>0.989</td>
</tr>
<tr>
<td>M 1.32</td>
<td>264.2**</td>
<td>256.6</td>
<td>250.02**</td>
<td>21.75</td>
<td>4.66</td>
<td>0.018</td>
<td>-0.025</td>
<td>1.026</td>
<td>1.100</td>
</tr>
<tr>
<td>Average values 20 mm</td>
<td>281.6</td>
<td>272.1</td>
<td>264.0</td>
<td>34.43</td>
<td>8.76</td>
<td>0.020</td>
<td>0.011</td>
<td>1.030</td>
<td>1.044</td>
</tr>
<tr>
<td>Average values for all samples</td>
<td>302.7</td>
<td>292.9</td>
<td>284.6</td>
<td>39.50</td>
<td>5.96</td>
<td>0.020</td>
<td>0.054</td>
<td>1.026</td>
<td>0.969</td>
</tr>
</tbody>
</table>

* mean value of yield strength of flanges lower than the manufacturer's declaration  
** design value of yield strength of flanges lower than the manufacturer's declaration
6. EFFECT OF YIELD STRENGTH ON BENDING RESISTANCE OF PLATE GIRDERS WITH CORRUGATED WEB

The presented chapter shows that the use of the yield strength for bending resistance of girders equal to the value of the specified minimum $R_{\text{min}}$ (coefficient $\gamma_m = 1$) makes it possible to obtain the resistance of the girders greater than that determined on the basis of the true design yield strength.

The performed tests showed that the mean yield strength $E(R_e)$ was greater than the guaranteed yield strength of delivered flat sheets that were used to prepare flanges for the tested corrugated web girders, and was equal to $R_{\text{min}} = 275$ MPa. This result was compatible with the methodology for assuming the specified minimum. However, the analysis of design yield strength of flanges in accordance with the methodology specified in EC3 [12] indicated a certain proportion of results that did not meet requirements for the minimum value. In the case of the discussed tests, 25% of them produced such results. Thus, there was an increase in the safe upper 5% limit state of safety quantile for structures related to bending resistance (Fig. 11a), and consequently, the likelihood of the structure failure was also higher (Fig. 11b).

The difference between bending resistance of the tested girders, which was reached due to the guaranteed yield strength $R_{\text{min}}$ for flange steel and the resistance determined from the true design yield strength $f_y$ is shown in Fig. 12. These results are presented separately for the girders with the simply supported beam scheme (Figs. 12a and b) and cantilever girders (Figs. 12c and d). Additionally, real use of bending resistance is presented in these Figures. However, the corrugated web girders were selected for tests to ensure that failure was determined by shear resistance. So, bending resistance was considerably lower than the allowable value.
EFFECT OF RANDOM STRENGTH PARAMETERS OF FLANGE STEEL ON BENDING RESISTANCE AND DEFLECTIONS OF GIRDERS...

For five tested girders, the obtained results for bending resistance that was determined on the basis of the specified minimum $R_{\text{min}}$, was greater than the true design yield strength $f_y$. The difference ranged from 1% (the girder M 2.32) up to 10% (the girder M 1.32). However, the difference was within the test range for the partial factor of yield strength $\gamma_m$. Thus, it is reasonable to specify this coefficient $\gamma_m = 1.1$ in national annexes as some EU states (e.g. France) do. It should be added that tension and compression resistance of flanges of corrugated web girders does not only depend on random variability of yield strength [28], but also on the arrangement of cross-sectional area of flanges in accordance with the following equations (17 and 18):

$$D^2(N_{t,Rd}) = \left(\frac{\partial N_{t,Rd}}{\partial A_t} \right)^2 D^2(A_t)^2 + \left(\frac{\partial N_{t,Rd}}{\partial R_c} \right)^2 D^2(f_y)^2,$$

(17)

$$D^2(N_{b,Rd}) = \left(\frac{\partial N_{b,Rd}}{\partial A_f} \right)^2 D^2(A_f)^2 + \left(\frac{\partial N_{b,Rd}}{\partial R_c} \right)^2 D^2(f_y)^2,$$

(18)

where: $N_{t,Rd}$ – resistance of the tension flange, $A_t$ – sectional area of the tension flange, $f_y$ – design yield strength of the element.

7. EFFECT OF THE YOUNG'S MODULUS $E$ ON DISPLACEMENTS OF PLATE GIRDERS WITH CORRUGATED WEB

The chapter illustrates the impact of the true modulus of Young obtained from material tests on the displacements of the tested girders containing end – plate connections. Considering the modulus of Young, it is crucial to use this quantity on the basis of its mean value determined from the tests. The total mean modulus of longitudinal deformation for all samples reached 206.1 GPa during the tests on flange samples from the girders. Thus, this result did not significantly differ from the value specified in the standard EC3 [19]. But values obtained for some samples were far from the mean value. This situation was described on the
example of the normal distribution of mean values of the longitudinal deformation modulus $E$ for individual girders with corrugated web (Fig. 13).

When the mean value of Young modulus for the whole samples population was used in the calculations, then the effect of displacements was negligible. Another situation was observed when the mean modulus determined for the given girder was used to estimate displacements. If this modulus was additionally combined with the effect of displacements resulting from the used semi-rigid connections, the allowable displacements specified in the standard [19] were expected to be exceeded.

The effect of modulus of Young on displacements was shown on two examples of the tested girders: M 1.11 (Fig. 2a) with the scheme of simply supported beam and M 1.12 (Fig. 2c) with the scheme of simply supported beam with the cantilever.

In the case of corrugated web girder M 1.11 with the scheme of simply supported beam, the overall displacement $y$ taking account of the effect of bending, shearing and rotation of two end-plate connections with stiffness $S_j$ was equal to [29]:

$$y = \frac{P_\alpha (3L^2 - 4a^2)}{24EJ_y} + \mu \frac{P_\alpha}{2GA} + \frac{2P_\alpha^2}{S_j},$$

(19)

where: $P$ – girder load, $I_y$ – modulus of inertia, $A$ – cross sectional area, $G$ – modulus of Kirchoff, $\mu$ – shear coefficient $= 0.85$, $L$, $a$ – dimensions of the girder in accordance with Fig. 2.

In the case of corrugated web girder M 1.12 with the scheme of simply supported beam with a single cantilever, the overall displacement $y$ taking account of the effect of bending, shearing and rotation of support butting profiles with stiffness $S_j$ was equal to [29]:

$$y = \frac{PL^2}{3EJ_y} (L + w) + \mu \frac{P_\alpha}{GA} + \frac{P_\alpha^2}{S_j},$$

(20)

where: $L$, $w$ – girder dimensions acc. to Fig. 2.

Rotational stiffness of connections in the tested girders were equal to 580 MNm/rad and 110 MNm/rad respectively. Displacements were determined for the critical load obtained from the tests [25].

Table 4 presents results for displacements that depend on the modulus of longitudinal deformation and rotational stiffness of connections $S_j$.

The effect of longitudinal deformation modulus $E$ on the true overall displacement of girders was within the range of 1.02-1.07. Thus, the modulus of longitudinal deformation $E$ did not significantly affect a change in displacements of corrugated web girders. The reason was that only flexural displacement was changed, and shear displacement or the displacement caused by rotation of connections were not affected. However, the correlation with improperly selected rotational stiffness of the connections could result in exceeding the allowable values that are strictly defined as 1/300 of the span length [24].

<table>
<thead>
<tr>
<th>Girder No. of sample</th>
<th>$y_1$ [for $S_j = 580$ MNm/rad, $E = 201.6$]</th>
<th>$y_2$ [for $S_j = 580$ MNm/rad, $E = 210$]</th>
<th>$y_3$ [for $S_j = \infty$, $E = 210$]</th>
<th>$y_1/y_2$</th>
<th>$y_1/y_3$</th>
<th>$\text{max}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 1.11</td>
<td>24.9</td>
<td>24.3</td>
<td>24.9</td>
<td>1.02</td>
<td>1.23</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>$y_1$ [for $S_j = 110$ MNm/rad, $E = 188.1$]</td>
<td>$y_2$ [for $S_j = 110$ MNm/rad, $E = 210$]</td>
<td>$y_3$ [for $S_j = \infty$, $E = 210$]</td>
<td>6.0</td>
<td>5.6</td>
<td>4.3</td>
</tr>
<tr>
<td>M 1.12</td>
<td>6.0</td>
<td>5.6</td>
<td>4.3</td>
<td>1.07</td>
<td>1.4</td>
<td>6.6</td>
</tr>
</tbody>
</table>
8. CONCLUSIONS

Bending resistance of corrugated web girders was affected by the quantile of the product of yield strength and random sectional area of the corrugated web.

When yield strength of ready flanges for corrugated web girders are defined, it is recommended to specify the relationships between the design yield strength and its mean value expressed as:

\[ f_y = E(R_e) - 3.04D(R_e), \tag{21} \]

The design yield strength related to the specified minimum value expressed below can be alternatively used to design of the structure made of corrugated web girders, which was confirmed by the conducted tests.

\[ f_y = R_{min}/\gamma_m \quad \text{and} \quad \gamma_m = 1.1, \tag{22} \]

Steel having similar yield strength is generally used to manufacture flanges of corrugated web girders.

Yield strength values of flange samples from the corrugated web girders from different batches were in 75% cases greater than the values declared by the manufacturer of corrugated webs, and in the remaining 25% cases these values were smaller. Thus, construction elements used at the construction site can have significantly diverged strength parameters.

Experimentally determined partial factor of yield strength \( \gamma_m \) values were within the range 1.01–1.05 and were comparable to factors determined from the statistical tests [13, 14, 15, 16]. These results were confirmed by values of bending resistance that were estimated on the basis of the specified minimum \( R_{min} \) and were higher than the true design yield strength \( f_y \) for 25% of tested corrugated web girders. The analysis of factors indicated that an increase in the coefficient \( \gamma_m \) to 1.1 is justified.

It should be mentioned that tests on metallurgical products do not include random arrangement of sectional thickness of the sample, which also affects bending resistance.

The effect of the modulus of Young \( E \) on true overall displacement of singular girders was within the range of 1.03–1.07. However, only 5% of the girder population was affected, which was an acceptable result for the whole statistical sample. The minor effect on the displacement of corrugated web girders was caused by the change that only affected flexural displacements with no impact on shear displacement and the displacement caused by rotation of the connections. However, the combined effect of the modulus of Young \( E \) and improperly selected rotational stiffness of the connections can result in exceeding the allowable displacement that are strictly defined as 1/300 of the span length.

ACKNOWLEDGEMENTS

The research is financed by the National Science Centre on the basis of the grant No. N N506 072538.

REFERENCES


