APPLICATIONS OF COMpressive SENSING OVER WIRELESS FADING CHANNELS

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Abstract-Wireless fading channels exist mixed noise, the most common noises are gaussian noise and impulse noise, to ensure the quality of the received signal, removing the noise in the channel is very important and necessary. WSDN (wavelet soft-threshold de-noising) can suppress low-intensity gaussian noise well; unfortunately, the denoising effect in removing impulse noise using WSDN is not obvious, especially when noise intensity is relatively high. BPDN (Basis Pursuit Denoising) which is a reconstruction algorithm of compressive sensing can control high-intensity gaussian noise in the channels well preceding WSDN, but also the effect of BPDN denoising the common impulse noise is not obvious, to denoise impulse noise, we adopt IBPDN (Improved Basis Pursuit Denoising) that changes L2 form to noise to L1 form, which was proposed in 2006 by Guosheng Bing. The experimental data show that IBPDN has good anti-noise ability to both gaussian noise and impulse noise; furthermore, more satisfactory results are obtained using IBPDN to mixed noise than those with BPDN.

Index terms: IBPDN, BPDN, WSDN, Wireless fading channels, denoising.
I. INTRODUCTION

Noise can be understood as the various factors to obstruct people or the system sensor to analysis the receive signal. The influence of noise on image processing is very big. It will affect every step such as input, transmission and output. Therefore, denoise or the research on the signal’s anti-noise property is an important step in image processing. The noise mainly discussed in this paper is gaussian noise and impulse noise.

In our research, we first analyze WSDN, we know WSDN [1] is commonly used in image denoising, however, wavelet denoising has some shortcomings, wavelet soft-threshold de-noising can’t restrain high intensity noise, moreover, the value of $\lambda$ is important for wavelet soft-threshold de-noising. Therefore, a more effective denoising method is essential to image denoising.

Recently, the theory of compressive sensing (CS) [2-5] has emerged as a promising approach that unifies signal sensing and signal compression into a single and simple task. The basic principle behind the CS framework is the use of nonadaptive linear projections to acquire an efficient, dimensionally reduced representation of a sparse signal. From that low-dimension representation, the original sparse signal can be recovered/reconstructed by solving an inverse problem [5]. Interestingly, the theory of compressive sensing has shown that by randomly projecting a sparse signal the most salient information is preserved in just a few measurements such that, with high probability, the sparse signal can be recovered from the measurements by solving the inverse problem $Y=AX+\xi$, where $Y$ is an M-dimension measurement vector, $A$ is the M*N measurement matrix with $M<<N$, $X$ is the target sparse signal, and $\xi$ is the noise vector. To the receiver, several algorithms has been proposed for signal reconstruction. Such as Matching Pursuit (MP) [6], Orthogonal Matching Pursuit (OMP) [7], Basis Pursuit (BP) [8] and Basis Pursuit Denoising (BPDN) [9-10] reconstruction algorithm.

Then, through further analysis, we obtain BPDN is a reconstruction algorithm, the basis pursuit algorithm for the signal’s sparsity is based on the L1 norm, but to the noise’s suppression, it adopted L2 norm constraint form, L2 norm can restrain gaussian noise very
well. What is more, as mentioned by Starck et.al., the advantages of this algorithm include the following: (a) There is no need to keep all the transform coefficients in memory, which is good especially for redundant transforms. (b) The algorithm has the capacity to include various constraints for optimization, which makes it flexible. The algorithm also automatically thresholds the coefficients for denoising purpose. And also BPDN can keep edges and details of image [11].

Images are transmitted through wireless fading channels, wireless fading channels exist mixed noise containing gaussian noise and impulse noise, and noise intensity is relatively high. Based on the above analysis, we conclude WSDN can control gaussian noise well, but it has bad anti-ability to high-intensity gaussian noise, and the denoising effect suppressing impulse noise is not obvious, Although BPDN can suppress high-intensity gaussian noise and is feasible, while it still exists some problems, its computational complexity is very high and the effect of denoising the common impulse noise is also not obvious, to denoise impulse noise, we consider changing L2 form to noise to L1 form, which was proposed in [12], L1 form’s optimization is a convex function optimization problem, and also the prior distribution of noise in L1 norm constraint is Laplace distribution, it is a kind of typical sparse distribution, suitable for inhibition of signal sparse noise.

The rest of this paper is organized as follows. WSDN and BPDN are illustrated in Section 2 and 3, the proposed method IBPDN is mainly focused in Section 4, then, Section 5 is a presentation of wireless fading channel. Corresponding experimental results are given in Section 6. Finally, Section 7 concludes this topic.

II. WAVELET SOFT-THRESHOLD DENOISING(WSDN)

During the experiment, the signals using the traditional way were analyzed and disposed with WSDN, and the signals are decomposed into two parts including approximation coefficient and detail coefficient by wavelet transforming. De-noising is a threshold quantization processing on the detail coefficients of each decomposition scale to select a threshold value, and then the signal is reconstructed. When the absolute value of wavelet coefficients is less
than the threshold, the wavelet coefficient is zero. Otherwise, the wavelet coefficient is the value of subtracting the threshold from them. It is showed in equation (1):

$$w_{i,t} = \begin{cases} 0, & |w| < \lambda \\ \text{sgn}(w)(|w| - \lambda), & |w| \geq \lambda \end{cases}$$  \hspace{1cm} (1)

When selecting threshold, the noise standard deviation $\sigma$ of original signal is determined with estimation method. It is showed in equation (2):

$$\sigma = \frac{\text{median}|w_i(k)|}{0.6745}$$  \hspace{1cm} (2)

Where, $i$ is wavelet decomposition scale, median is the order of the value. Unified threshold of Donoho and Johnstone is adopted in this paper.

$$\lambda = \sigma \sqrt{2 \log N}$$  \hspace{1cm} (3)

In which, $N$ is the length of signal.

WSDN is an appealing denoising technique, but, it has some problems, firstly, the value of $\lambda$ is important for wavelet soft-threshold de-noising, then, WSDN can’t be used to remove high intensity noise. To overcome these problems, we should employ the CS method.

### III. BASIS PURSUIT DENOISING (BPDN)

The no noise image is sparse in the wavelet domain. It can directly acquire a compressed signal representation using the CS method. After transmit the sampled signal, we can get the reconstruction signal by some kinds of nonlinear optimization reconstruction method such as basis pursuit algorithm in the receiver. However, when the sampled signal transmitted, there will be some noise inevitably. Now, we will mainly discuss the sampled signal’s reconstruction property after through the fading channels.

The origin signal can be described as:

$$x = \sum_{i=1}^{N} \alpha_i \psi_i = \Psi \alpha$$  \hspace{1cm} (4)

Where, $x$ is the origin signal, $\psi$ is based-wavelet, $\alpha$ is the projected coefficient the length of $x$) coefficients. so $\psi$ is the sparsity domain for $x$, $x$ is K-sparsity on $\psi$. 

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From the compressive sensing theory, if \( x \) is K-sparsity, we can consider a general linear measurement process that computes \( M < N \) inner products between \( x \) and a collection of vectors \( \{ \varphi_j \} (j=1\ldots M) \) as in \( y_j = \langle x, \varphi_j \rangle \). Arrange the measurements \( y_j \) in an \( M \times 1 \) vector \( y \) and the measurement vectors \( \varphi_j^T \) as rows in an \( M \times N \) matrix \( \Phi \). Then, by substituting \( \Phi \) from equation (4), \( y \) can be written as:

\[
y = \Phi x = \Phi \varphi \alpha
\]

Where, \( y \) is the measurement value, \( \Phi \) is the measurement matrix. The measurement process is not adaptive, meaning that \( \Phi \) is fixed and does not depend on the signal \( x \).

After the sampled signal, that is the measurement value transmitted in the fading channels, \( y_{ni} \) can be written as:

\[
y_{ni} = \Phi x + z = \Phi \varphi \alpha + z, \quad \|z\| \leq \xi
\]

Where, \( z \) is the noise, \( y_{ni} \) is the data to transmit.

In the receiver, we use BPDN algorithm to solve the origin signal. BPDN define the optimization problem as equation (9):

\[
\min \| \alpha \| \quad \text{s.t.} \quad \| y_{ni} - \Phi \varphi \alpha \|_2 \leq \xi
\]

Where, \( \xi \geq \|z\|_2 \) is the allowed maximum error. Equation (8) refers to the solution of:

\[
\min_{\alpha} \frac{1}{2} \| y_{ni} - \Phi \varphi \alpha \|_2^2 + \lambda \| \alpha \|
\]

The solution \( \alpha(\lambda) \) is the function of parameter \( \lambda \). It yield a decomposition into the signal-plus-residual:

\[
y = s(\lambda) + r(\lambda)
\]

Where, \( s(\lambda) = \Phi \alpha(\lambda) \). The size of the residual is controlled by \( \lambda \). As \( \lambda \rightarrow 0 \), the residual goes to zero and the solution behaves exactly like BP. As \( \lambda \rightarrow \infty \), the residual gets large; the \( r(\lambda) \rightarrow y \) and \( s(\lambda) \rightarrow 0 \).

It has been proved that equation (8) is equivalent to the following perturbed linear program [13-16] :

\[
\min c^T x + \frac{1}{2} \| \rho \|_2^2 \quad \text{subject to} \quad Ax + \delta p = b, x \geq 0, \delta = 1
\]
Where $A=(\Phi, -\Phi); b=y; c=1$. Perturbed linear programming is really quadratic programming, but retains structure similar to linear programming. We can solve equation (10) to find the optimal solution from $y_{ni}$, and get the reconstruction signal through $y_{ni}$. Parameter $\lambda$ compromised between the signal’s residual and coefficient sparse degree, and it’s very important to the reconstruction signal’s quality. Reference set $\lambda = \xi \sqrt{2 \log (N)}$ to be the best value. Where, $N$ is the length of the signal, $\xi$ is the maximum permissible error.

IV. IMPROVED BASIS PURSUIT DENOISING (IBPDN)

Although BPDN is feasible, it still exists two problems, the first one is that this method’s computational complexity is very high, and the other one is that BPDN only performs well in gaussian noise channels, but in fact noise is diverse, the denoising effect of BPDN denoising the common impulse noise is not obvious. In view of the above problems, someone put forward the improved basis pursuit denoising algorithm.

BPDN reflecting signal’s sparse is based on L1 norm, and adopted L2 form to control noise, as we know, L2 form can not reflect signal’s sparse [17], so it can’t suppress the sparsity noise well, so also to impulse noise, to denoise impulse noise, we should modify the constraint condition to noise, firstly, we consider $p(p>2)$ form constraint condition, but because the optimization of $p(p>2)$ form is a convex function optimization problem, in which there are a lot of mathematical problems [18]. While L1 form’s optimization is a convex function optimization problem, and also the prior distribution of noise in L1 norm constraint is Laplace distribution, it is a kind of typical sparse distribution [19], suitable for inhibition of signal sparse noise. So, we change L2 form to L1 form, as follow

$$\min_j \| z_j \|_{s.t.} \| y_{ni} - \Phi \varphi \|_1 \leq \xi$$

Equation (11) refers to the solution of:

$$\min_{\alpha} \frac{1}{2} \| y_{ni} - \Phi \varphi \alpha \|_1 + \lambda \| \alpha \|_1$$

It has been proved that equation (12) is equivalent to the following perturbed linear program [13-16]:
\[
\min c^T x \quad \text{subject to} \quad Ax = b
\]  
(13)

Where \( A = (I, I, \Phi, -\Phi) \), \( c = \left( \frac{1}{\lambda} \right) \), \( b = y \), and solving the linear program (13), we can see the literature [20].

V. WIRELESS FADING CHANNEL

In this section, we firstly analyze the transmission principle using the three methods and the characteristics of noisy channels, then, we combine the three methods with the characteristics of noise to analyze these methods' denoising ability, through above analysis, we obtain that IBPDN has good anti-ability to both gaussian noise and impulse noise preceding WSDN and BPDN.

When denoising image \( y \) WSDN, we transmit the original image directly via fading channels. When we use BPDN and IBPDN to denoise image, we transmit the measurement image. The system chart is as follows

![System chart](image1)

Figure 1 the wireless image transmission and reconstruction

a. Gaussian fading channel

The envelope of the fading channel is gaussian distributed with following pdf (probability density function):

\[
p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-u)^2}{2\sigma^2}}
\]

(14)

Where \( x \) is the envelope and \( u \) is the average value, \( \sigma^2 \) is the variance.

b. Impulse fading channel

In short, the envelope of the fading channel is impulse distributed with following pdf:
If \( b > a \), gray value \( a \) in noisy image will be shown as a dark dot, while, \( b \) will be shown as a bright dot. If \( p_a \) or \( p_b \) is 0, the pulse noise will be unipolar pulse, if \( p_a \) and \( p_b \) are impossible to be 0, especially when the \( p_a \approx p_b \), impulse noise value will be similar to pepper and salt random distributing in the image.

We know the curve of gaussian distribution is continuous. When noise intensity increases, then the variance of noise will accordingly increase, the curve of pdf will become smoother when the variance of noise increase, which leads to the probability larger of points that are less than the maximum point or more than the maximum point, it means that the points of noise mainly distribute in the two sides of the maximum point. To WSDN, this phenomenon is bad, because \( \lambda \) is very difficult to choose, \( \lambda \) must be enough large which will make image information miss, so WSDN has bad suppression to high-strength noise. Although in term of BPDN, which adopted L2 norm constraint form to suppress noise, which is effective to minimize continuous function, so, L2 norm can restrain gaussian noise. But, L2 norm cannot suppress impulse noise very well, because L2 norm can’t reflect signal’s sparsity, so to the sparsity noise.

We also know the pdf of impulse noise is uncontinuous, and the noise is sparse, thus, BPDN has a bit bad anti-noise ability to this noise, what is more, the wavelet coefficients is very large to image wavelet coefficients when impulse noise is expanded into its wavelet coefficients, to denoise impulse noise, \( \lambda \) must be enough large which will make image information miss, so WSDN also has bad anti-noise ability to this noise, the proposed method IBPDN in this paper adopt L1 norm to control noise, L1 norm can reflect noise’s sparsity, so, IBPDN can eliminate impulse noise well, so also to gaussian noise.

The wireless transmission system discussed here is distributed in a large area and the transmission distance is relatively far, then the noise intensity will be high and noise is diverse, so IBPDN is effective, a series of simulations will be present in the next section to prove IBPDN is effective to high intensity gaussian noise and impulse noise.
VI. IMAGE RECOVERY AND DENOISING

We now present some experimental results to compare our IBPDN, WSDN, with the BPDN in different noise environment, different noise intensity and different $\lambda$ values when noise intensity is relatively high or low. As usual we use the value of PSNR (peak signal-to-noise ratio) to measure the quality of a restored image: if $f(x, y)$ is the original image, $\tilde{f}(x, y)$ the restored one, then

$$PSNR(\tilde{f}) = 10 \log_{10} \left( \frac{255^2}{MSE} \right)$$

$$MSE = \frac{1}{I} \sum_{i=1}^{I} (\tilde{f}(x, y) - f(x, y))^2$$

The larger is the value of PSNR, the better is the quality of restoration. All images used in our experiments are Lena images of size $256 \times 256$.

This paper will analyze how $\lambda$ affect the denoising effect in different noise environment, we firstly analyze the denoising effect of the three methods in the case of low noise intensity, if the proposed method outperforms the other two methods in controlling noise, then, we will next compare the three methods in more severe situation, that is high noise intensity.

a. The denoising effect when $\lambda$ changes (noise intensity is small)

In this part, we will analyze in low intensity noise environment, how the change of $\lambda$ affects the denoising effect, we take 30db gaussian noise and 0.3 impulse noise for an example.

In a relatively low intensity noise channel in which impulse noise intensity is 0.3 and gaussian noise intensity is 30dbw, we compare the three methods with different value of $\lambda$. In figure 2(a), the noise in the channel is gaussian noise, from figure 2(a), we can know that for WSDN method, when $\lambda$ is increased to a defined value (about 50), the denoising effect is significantly improved, when $\lambda$ is more than this value, there will be no obvious change, in theory, through the formula (3), we can calculate $\lambda$ here is 150, so we set the $\lambda$ to be 50 to 350. In figure 2(b), the noise in the channel is impulse noise, based on the above analysis, we get $\lambda$ here is 347, thus we set the $\lambda$ to be 100 to 900.
Figure 2 Comparison among the three methods with different $\lambda$ (a) The psnr of the reconstructed gaussian noisy image (gaussian noise is 30dbw) (b) The psnr of the reconstructed impulse noisy image (impulse noise is 0.3)

In figure 3, we restore the noisy image using WSDN, BPDN and IBPDN in a 30db gaussian noise channel, we choose the $\lambda$ is 50, in figure 4, we compare IBPDN with BPDN, WSDN in removing a 0.3 impulse noise when $\lambda$ is 300.

Figure 3 (a) The original image Lena; (b) the gaussian noisy image (noise intensity is 30dbw) (c) Image a restored by WSDN($\lambda=50$) (d) Image a restored by BPDN($\lambda=50$) (e) Image a restored by IBPDN($\lambda=50$)
The experiment results show that the PSNR using the three methods all increase when $\lambda$ rises when noise intensity is lower, and IBPDN and BPDN clearly outperform WSDN in removing gaussian noise and impulse noise, furthermore IBPDN has slightly better anti-noise ability than BPDN, we also get the denoising effect using WSDN relatively is not obvious when $\lambda$ is minore, and the PSNR using WSDN is relatively stable when $\lambda$ is more than 150 or 347, while the PSNR using the other two methods are stable in the whole $x$ axis. Because in low noise situation, IBPDN has better anti-noise ability, so we should do the next experiment.

b. The denoising effect when $\lambda$ changes (noise is relatively strong)

In this part, we will analyze in high intensity noise environment, how the change of $\lambda$ affects the denoising effect, we take 50db gaussian noise and 0.5 impulse noise for an example.

In figure 5, similarly we also compare our IBPDN with BPDN, WSDN in removing a relatively higher 50db intensity gaussian noise and a relatively higher 0.5 intensity impulse noise with different value of $\lambda$. Figure 5(a) shows the noise in channel is gaussian noise, for WSDN method, when $\lambda$ is increased to a defined value, the denoising effect is significantly improved, when $\lambda$ is more than this value (about 1000), there will be no obvious change, in theory, through the formula (3), we can calculate the default value of the $\lambda$ is 1478, so we set the $\lambda$ in the example to be 500 to 3500. Figure 5(b) shows the noise in channel is impulse noise,
and the default value of the $\lambda$ is 457, so we set the $\lambda$ to be 100 to 900.

![Figure 5](image5.png)

Figure 5 Comparison among the three methods with different $\lambda$ (a) The psnr of the reconstructed gaussian noisy image (gaussian noise is 50dbw) (b) The psnr of the reconstructed impulse noisy image (impulse noise is 0.5)

In figure 6, we reconstruct the noisy image using WSDN, BPDN and IBPDN in a 50db gaussian noise channel, and the $\lambda$ is 500, in figure 7, we compare IBPDN with BPDN, WSDN in removing a 0.5 impulse noise when $\lambda$ is 300.

![Figure 6](image6.png)

Figure 6 (a) The original image Lena; (b) the gaussian noisy image (noise intensity is 50dbw) (c) Image a restored by WSDN($\lambda=500$) (d) Image a restored by BPDN($\lambda=500$) (e) Image a restored by IBPDN($\lambda=500$)
From this experiment, we can also obtain that IBPDN and BPDN are much more efficient than WSDN in suppressing gaussian noise and impulse noise, and IBPDN control noise slightly better than BPDN when noise increases to a relatively high intensity.

Conclusion: no matter the noise intensity is large or small, under the same $\lambda$ value, BPDN and IBPDN have better anti-noise ability than WSDN. Then in the following experiment, we will fix the $\lambda$ value, and analyze the denoising effect when noise intensity changes.

c. The denoising effect when noise intensity changes

To analyze the noise intensity effect on reconstructing noisy image, in the third experiment, we compare the three methods with different noise intensity, in figure 8(a), we set gaussian noise intensity to be 10db,12db,14db,.....50db and in figure 8(b), impulse noise intensity to be 0.1,0.2,0.3,......0.6, $\lambda$ is the default value.
Figure 8 comparison among the three methods with different noise intensity, $\lambda$ is default value.

(a) The psnr of the reconstructed gaussian noisy image
(b) The psnr of the reconstructed impulse noisy image

In figure 9, we reconstruct the noisy image using WSDN, BPDN and IBPDN in a 40db gaussian noise channel, and the $\lambda$ is the default value. In figure 10, we compare IBPDN with BPDN, WSDN in removing a 0.4 impulse noise, the $\lambda$ is also default value.

Figure 9 (a) The original image Lena; (b) the gaussian noisy image (noise intensity is 40dbw)
(c) Image a restored by WSDN (d) Image a restored by BPDN (e) Image a restored by IBPDN
The results show that when noise intensity changes and $\lambda$ value is fixed, IBPDN and BPDN still has better anti-noise ability, WSDN only can suppress relatively lower gaussian noise (less than 20db), and is not suitable for impulse noise.

The above three experiments show no matter $\lambda$ value changes or not, both BPDN and IBPDN can remove the single noise better than WSDN in wireless fading channels, and IBPDN has slightly better anti-ability than BPDN. But, in fact, channels not just exist one kind of noise, maybe are often affected by two kinds of noise at the same time, so in the following experiment, we will analyze in mixed noise conditions, the denoising effect of BPDN and IBPDN.

d. The denoising effect in removing different intensity mix noise

From the front experiments, we know that IBPDN and BPDN can restrain noise much better than WSDN, so, in the forth experiment, we only compare IBPDN and BPDN in removing different intensity mix noise containing gaussian noise and impulse noise, the PSNR values are presented in table 1, the restored images are shown in figure 11 in which gaussian noise intensity 20db and impulse noise intensity is 0.2, the results show that IBPDN clearly outperforms BPDN in removing a mixed noise, both visually and quantitatively.

Figure 10 (a) The original image Lena; (b) the impulse noisy image (noise intensity is 0.4) (c) Image a restored by WSDN (d) Image a restored by BPDN (e) Image a restored by IBPDN
Table 1 PSNR values in removing mix noises with BPDN & IBPDN

<table>
<thead>
<tr>
<th>Noisy images</th>
<th>10db+0.1</th>
<th>20db+0.2</th>
<th>30db+0.3</th>
<th>40db+0.4</th>
<th>50db+0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR by BPDN</td>
<td>29.2143</td>
<td>29.0684</td>
<td>28.9284</td>
<td>28.7164</td>
<td>28.4626</td>
</tr>
<tr>
<td>PSNR by IBPDN</td>
<td>31.5757</td>
<td>31.533</td>
<td>31.4703</td>
<td>31.2585</td>
<td>31.0985</td>
</tr>
</tbody>
</table>

From the above experiments, we can know BPDN and IBPDN always have much better anti-noise ability than WSDN, both visually and quantitatively, especially when the noise is relatively high. We can also get that BPDN and IBPDN both have good suppression ability to gaussian and impulse noise, furthermore, IBPDN has a little better suppression ability than BPDN in gaussian or impulse noise. From Table 1 and figure 11, IBPDN has better anti-noise ability than BPDN to mix noise, the PSNR using the IBPDN method increases 2db to BPDN. Thus, IBPDN can be used to suppress noise in wireless fading channel.

VII. CONCLUSIONS

In this paper, we compare WSDN, BPDN with IBPDN in wireless fading channels, experiments verify that the basis pursuit has good suppression to high-strength gaussian noise, and IBPDN can suppress both noise very well, while WSDN only has good anti-noise ability when noise intense is relatively low. What is more, IBPDN has a little better anti-noise ability than BPDN to mixed noise, In sum, the proposed method exposes significant dominance on the fading channels and has better performance for different noise distributions. Although, here the noise we discussed is transmission noise, in experiments, we add noise after the measurement, and noise not only exist in channels, but also in the process of image.
acquisition, so, the future job is to improve BPDN so that it can restrain the mix noise in the process of image acquisition very well.

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