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Resource allocation for NOMA based networks using relays: cell centre and cell edge users

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Abstract

Nonorthogonal multiple access (NOMA) transpire out as a solution to revamp the problem of spectral efficiency, allowing some level of interference at receivers. Recently, relays are utilized to improve access of cell edge users. The utilization of relays improves spectral efficiency with reduced outage probability. In this paper, the relays used have the capability of performing successive interference cancellation (SIC) for the users connected to it and regenerates only the signals of the users connected to it. The cell edge users are accessible to the base station in an environment where multiple relays are available, and where the user selects the link with the best channel quality. The user's mobility is also considered during time sub-slot and used while obtaining the user's ergodic rate and outage probability in the presence of a higher signal to noise ratio. Simulation results are used to show the performance improvement of the proposed method as compared to available work in literature.

Keywords
NOMA, Outage probability, Ergodic sum rate, SIC, HetNets.

There is a severe strain on the current mobile communication system due to the significant increase in users and wireless applications. Each with its own set of transmission requirements, which needs to be satisfied by the communication system. This requirement is difficult to achieve with the current orthogonal multiple access technologies as it cannot meet the increased data demand. It falls short in a few performance areas such as spectral efficiency, user fairness, and compatibility. The system experiences low spectral efficiency due to the utilization of OFDMA, which does not consider the quality of the channel condition of each user when assigning resources. The lack of user fairness is a result of priority scheduling based on the user's channel conditions (Zhao et al., 2019; Al-Abbasi and So, 2016; He et al., 2016; Cheng et al., 2015; Aldababsa et al., 2018).

A multiple access technique, called Non-Orthogonal Multiple Access (NOMA) has been proposed to accommodate the increased data demands, by improving the sum-rate and spectral efficiency of the system (Fang et al., 2017; Xu and Cumanan, 2017). NOMA utilizes the power domain to service multiple users simultaneously by multiplexing them over the same resource; however, with varying power levels (Choi, 2017; Yang et al., 2016).

The analysis of secondary users in a NOMA base cognitive radio is explored in the study of Balyan (2020), whereby the NOMA power allocation is used. The users and system outage probability is derived and various parameters are used in the performance analysis.

NOMA implements superposition coding (SC) and successive interference cancellation (SIC) at the transmitter and receiver, respectively (Timotheou and Krikidis, 2015; Oviedo and Sadjadpour, 2018), and capitalizes on the varying channel conditions of the users.

NOMA has been the main focus in recent research, where crucial performance parameters of the wireless communication system have been explored. The ergodic capacity of a NOMA implemented MIMO system with transmitter channel state information is explored in the study of Sun et al. (2015), where two power allocation schemes maximize the ergodic capacity of the system. The first PA scheme utilizes an algorithm to allocate power optimally but is very complicated. Thus, to reduce the computational complexity, the authors present a sub-optimal power allocation scheme. Simulations results give the superior performance of both PA schemes in comparison to OMA. In the study of Manglayev et al. (2016), an improvement in the system throughput with user fairness using a NOMA based power allocation scheme is reported. Like Manglayev et al. (2016), the results of a downlink NOMA system in Di et al. (2016) shows an improvement in user fairness and total sum-rate when compared to OFDMA. It explores a joint scheme to maximize the downlink sum-rate of the single-cell NOMA system, where iterative algorithms based on a matching game concept perform the assignment of sub-channels and allocation of power. In the study of Fang et al. (2017), scheduling and power allocation of users is used to derive the sum-rate, which in turn is used to obtain the energy efficiency of the system. However, this results in a non-convex optimization problem. A separation of the user scheduling and power allocation into two individual problems results in the transformation of the optimization problem into a convex problem. After which, two separate user scheduling and power allocation algorithms perform the resource allocation.

The maximization of the energy efficiency of a hybrid downlink NOMA-OFDM system is the focus in the study of Shi et al. (2019) to capitalize on the advantages of each. They propose two resource allocating algorithms, one to optimally allocate resources and one low complexity resource allocating algorithm. Each algorithm contains two power allocating algorithms, one where the power consumption is limited and the other not, respectively. Results show the performance of the proposed schemes outperforms conventional OMA schemes. According to Ding et al. (2016), their downlink MIMO-NOMA system performs better than conventional MIMO-OMA in terms of power allocation coefficients selection. A system where all users have a fixed power allocation provides a baseline for the proposed precoding scheme, where simulation results show the superiority of the scheme in terms of outage probability. In addition to this, the authors implement a scheme to pair users with different channel conditions. Then two quality of service constraints is the basis of the power allocation coefficient selection. An antenna selection scheme for the downlink transmission of a MISO-NOMA channel is the focus in the study of Shrestha et al. (2016). The transmit antenna selection (TAS) is employed by the BS, where the antenna selection depends on the best sum-rate from the group of transmit antennas. A TAS-NOMA algorithm evaluates the sum rates of each antenna and selects the highest sum rate. According to the results, the utilization of this scheme results in an improved sum-rate.

In the study of Zeng et al. (2019), the uplink transmission of a NOMA implemented millimeter-wave massive MIMO system is the area of investigation. The channel conditions of users determine their cluster groupings, clusters consist of two users. A beamforming technique applied at the BS follows the application of NOMA to each cluster. After which an algorithm based on the quality of service performs the allocation of power, which results in the maximization of energy efficiency. However, the authors go a step further by using the SINR to remove the inter-cluster interference.

Results show that both schemes, with and without the removal of inter-cluster interference, perform better than OMA. In the study of Nasser et al. (2019), heterogeneous networks (HetNets) is explored, where they investigate the downlink transmission of a NOMA based MIMO system. Stage 1 of a two-stage scheme focuses on the inter-cluster and co-tier interference of small-cell and macro-cell tiers, while stage 2 utilizes game tactics to perform the power allocation. Two separate algorithms execute stages 1 and 2, interference alignment and coordinated beamforming (IA-CB) and non-cooperative game-based power allocation, respectively. The proposed scheme outperforms MIMO-OMA and MIMO-NOMA HetNets concerning sum-rate and outage probability.

Minimizing the power consumption of the base station is the primary objective in the study of Bonnefoi et al. (2019). They utilize the benefits of NOMA and cell discontinuous transmission (Cell DTx) to design a power allocation scheme. With the implementation of Cell DTx, the BS employs two modes of operation, active and sleep mode, respectively. During active mode, the BS utilizes NOMA to aid users for a specific period, after which it enters sleep mode, which is an energy conservation state. The authors use the active state of the BS to derive the power allocation expressions, after which the utilization of KKT conditions result in optimal power allocation. A joint approach by Zhang et al. (2019) explore a MIMO-NOMA system where the effective capacity (EC) forms the basis of resource allocation. First, users are grouped into clusters using a CSI-based algorithm; then, a beamforming vector cancels inter-cluster interference. Two algorithms perform the channel and power allocation for a fixed power and channel allocation, respectively. A final algorithm performs the optimization of the EC-based power and channel allocation. Results show that the proposed algorithms are less complicated in comparison to traditional algorithms. Yuan et al. (2019) develop a proportional rate constraint-based power allocation algorithm to increase the energy efficiency of a multi-carrier NOMA system, the derivation of the power allocation to maximize the energy efficiency results in a non-convex optimization problem. Due to the non-convexity, the problem is divided into two and solved individually; then, an algorithm performs the power allocation. According to results, the EE and SE outperform conventional OMA.

System model

The proposed model for the system is shown in Figure 1 which has one BS, three relays denoted by r_i, i = 1, 2, 3 and four random users denoted as UE_j, j = 1, 2, 4. The users change their positions from cell center users to cell edge users. The cell center users communicate directly with the BS, while the cell edge users use the relays for communication with the BS as they operate in half-duplex mode. The h_j^{BS} denotes the channel coefficient from the BS to user j and h_j^r denotes the channel coefficient from a relay to user j. The channels are independent and are under the influence of Rayleigh fading. The channels are modeled as $h_j^{BS} \sim CN\left(0, \lambda_j^{BS}\right)$. The cell center users' channel conditions are better than the cell edge users. For cell edge users, a selected relay is used to forward signals from the BS using NOMA. As shown in Figure 1, a UE can be in the coverage area of one or more relays. A cell edge UE is connected to the BS in two hops, the communication link between the BS and relay is the relay link, and the communication link between a relay and UE is the access link. The UE selects the relay with the best channel coefficient in a specific time slot. The smartly equipped relays used in this paper can perform SIC by removing information signals of UEs not connected to it. In doing so, the relay avoids the unnecessary regeneration of signals for higher transmission power UEs. Thus, without this regeneration of other UEs signals, the relays can provide better service to the UEs connected to it.

Further, the time slot is divided into four sub-slots denoted by t_{s_k}, $k = 1, 2, ..4$ and $t = \sum_{k=1}^{4} t_{sk}$. The channel conditions in a time sub-slot remain constant. All the time sub-slots are equal i.e. $t_{s1} = t_{s2} = t_{s3} = t_{s4}$.

Time sub-slot t_{s1}

Let UE_1, UE_2, UE_3 and UE_4 be active users. The status of these users according to their locations as shown in Figure 1, is (a) UE_1 and UE_2 are cell center user, (b) UE_3 and UE_4 are cell edge users. The channel conditions of the access link between UE_4 and relays r_3, r_2 and r_1 are worst, better, and best respectively, thus UE_4 chooses r_1 as its access link to the BS. The channel conditions of the access link between UE_3 and relays r_1, r_3 and r_2 are worst, better and best respectively, thus UE_3 chooses r_2 as its access link to the BS.

The superimposed signal UE_1, UE_2, UE_3, and UE_4 transmitted by the BS to the connected user equipment's UE_1, UE_2 relays r_1 and r_2 is:

$$x_s\left(t_{s_1}\right) = \sqrt{P_1^{t_{s_1}} P_s}\, x_1\left(t_{s_1}\right) + \sqrt{P_2^{t_{s_1}} P_s}\, x_2\left(t_{s_1}\right) + \sqrt{P_3^{t_{s_1}} P_s}\, x_3\left(t_{s_1}\right) + \sqrt{P_2^{t_{s_1}} P_s}\, x_4\left(t_{s_1}\right) \quad (1)$$

where $x_1\left(t_{s_1}\right), x_2\left(t_{s_1}\right), x_3\left(t_{s_1}\right)$ and $x_4\left(t_{s_1}\right)$ are UE_1, UE_2, UE_3, and UE_4 data symbols with expected value equal to 1. The power allocation of BS is denoted by P_s, the power allocation coefficients for UE_1, UE_2, UE_3, and UE_4 are $P_1^{t_{s_1}}, P_2^{t_{s_1}}, P_3^{t_{s_1}}$ and $P_4^{t_{s_1}}$, also $P_1^{t_{s_1}} + P_2^{t_{s_1}} + P_3^{t_{s_1}} + P_4^{t_{s_1}} = 1$ with $P_1^{t_{s_1}} < P_2^{t_{s_1}} < P_3^{t_{s_1}} < P_4^{t_{s_1}}$. The received superimposed signals at UE_1, UE_2, r_1 and r_2 are given as:

$$\begin{aligned} y_1\left(t_{s_1}\right) &= h_1^{BS} x_s\left(t_{s_1}\right) + N_1 \\ y_2\left(t_{s_1}\right) &= h_2^{BS} x_s\left(t_{s_1}\right) + N_2 \\ y_{r_1}\left(t_{s_1}\right) &= h_{r_1}^{BS} x_s\left(t_{s_1}\right) + N_{r_1} \\ y_{r_2}\left(t_{s_1}\right) &= h_{r_2}^{BS} x_s\left(t_{s_1}\right) + N_{r_2} \end{aligned} \quad (2)$$

where N_r denotes the AWGN at respective user or relay.

Each UE must perform SIC on the received superimposed signal and decode the signals stronger than itself to extract its signal of interest. The signal with the strongest power or the signal with the worst channel conditions are decoded first. In this case, UE_1 first decodes the signals for UE_4, UE_3 and then UE_2. The signal to interference and noise ratio ($SINR$) of the decoded signals are:

$$\begin{aligned} SINR_{t_{s_1}}^{4-1} &= \frac{P_4^{t_{s_1}} P_s \left|h_1^{BS}\right|^2}{\sum_{j=1}^{3} P_j^{t_{s_1}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s_1}}^{3-1} &= \frac{P_3^{t_{s_1}} P_s \left|h_1^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s_1}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s_1}}^{2-1} &= \frac{P_2^{t_{s_1}} P_s \left|h_1^{BS}\right|^2}{P_1^{t_{s_1}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \end{aligned} \quad (3)$$

After removing the decoded SINR of UE_2, UE_3, and UE_4, the decoded SINR of the received signal for UE_1 is:

$$SINR_{t_{s_1}}^{1-1} = \frac{P_1^{t_{s_1}} P_s \,|\, h_1^{BS} \,|^2}{\sigma^2} \tag{4}$$

The data rate of the UE_1 is:

$$t_{s_1} log_2 \left(1 + \frac{P_1^{t_{s_1}} P_s \,|\, h_1^{BS} \,|^2}{\sigma^2} \right) \tag{5}$$

The throughput achieved by UE_1 is:

$$Th_1 = n_{rb1} B \, log_2 \left(1 + \frac{P_1^{t_{s_1}} P_s \,|\, h_1^{BS} \,|^2}{\sigma^2} \right) \tag{6}$$

where n_{rb1} denotes the number of resource blocks assigned, of bandwidth B, and σ^2 is the AWGN variance.

After receiving the superimposed signal, UE_2 performs SIC for UE_3 and UE_4. It first decodes the signals for UE_4 and then UE_3:

$$\begin{aligned} SINR_{t_{s_1}}^{4-2} &= \frac{P_4^{t_{s_1}} P_s \left| h_2^{BS} \right|^2}{\sum_{j=1}^{3} P_j^{t_{s_1}} P_s \left| h_2^{BS} \right|^2 + \sigma^2} \\ SINR_{t_{s_1}}^{3-2} &= \frac{P_3^{t_{s_1}} P_s \,|\, h_2^{BS} \,|^2}{\sum_{j=1}^{2} P_j^{t_{s_1}} P_s \,|\, h_2^{BS} \,|^2 + \sigma^2} \end{aligned} \tag{7}$$

After removing the decoded SINR of UE_3 and UE_4, the decoded SINR of the received signal of UE_2 with interference from UE_1 still present is:

$$SINR_{t_{s_1}}^{2-2} = \frac{P_2^{t_{s_1}} P_s \,|\, h_2^{BS} \,|^2}{P_1^{t_{s_1}} P_s \,|\, h_2^{BS} \,|^2 + \sigma^2} \tag{8}$$

The minimum SINR of UE_2 is:

$$a_2^{t_{s_1}} = \min\left(SINR_{t_{s_1}}^{2-2}, SINR_{t_{s_1}}^{2-1} \right) \tag{9}$$

The data rate of the UE_2 in time sub-slot t_{s_1} is:

$$t_{s_1} log_2 \left(1 + a_2^{t_{s_1}} \right) \tag{10}$$

The throughput achieved by UE_2 is:

$$Th_2 = n_{rb2} B \, log_2 \left(1 + a_2^{t_{s_1}} \right) \tag{11}$$

The relay r_1 decodes the signal of UE_4 directly with interference from UE_1, UE_2, and UE_3 still present. The decoded SINR at the relay r_1 is:

$$SINR_{t_{s_1}}^{4-r_1} = \frac{P_4^{t_{s_1}} P_s \left| h_{r_1}^{BS} \right|^2}{\sum_{j=1}^{3} P_j^{t_{s_1}} P_s \left| h_{r_1}^{BS} \right|^2 + \sigma^2} \tag{12}$$

The relay r_2 decodes the signal of UE_4 and then removes the decoded signal to obtain the signal of interest for UE_3 with interference from UE_1 and UE_2 and still present. The decoded SINR at the relay r_2 is:

$$\begin{aligned} SINR_{t_{s_1}}^{4-r_2} &= \frac{P_4^{t_{s_1}} P_s \left| h_{r_2}^{BS} \right|^2}{\sum_{j=1}^{3} P_j^{t_{s_1}} P_s \left| h_{r_2}^{BS} \right|^2 + \sigma^2} \\ SINR_{t_{s_1}}^{3-r_2} &= \frac{P_3^{t_{s_1}} P_s \left| h_{r_2}^{BS} \right|^2}{\sum_{j=1}^{2} P_j^{t_{s_1}} P_s \left| h_{r_2}^{BS} \right|^2 + \sigma^2} \end{aligned} \tag{13}$$

Time sub-slot t_{s2}

In this slot, the relays regenerate the new signals.

The signals generated by r_1 and r_2 are:

$$x_{s_{r_2}}\left(t_{s_2}\right) = \sqrt{P_3^{t_{s_2}} P_{r_2}} \, x_3\left(t_{s_1}\right) \tag{14}$$

$$x_{s_{r_2}}\left(t_{s_2}\right) = \sqrt{P_3^{t_{s_2}} P_{r_2}} \, x_3\left(t_{s_1}\right) \tag{15}$$

The power allocation of relays are denoted by P_{r_1}, and P_{r_2}, and the power allocation coefficients for UE_3 and UE_4 are $P_3^{t_{s_2}}$ and $P_4^{t_{s_2}}$. The received signal at the UE_3 and UE_4 are given as:

$$\begin{aligned} y_{r_2}\left(t_{s_2}\right) &= h_3^{r_2} x_{s_{r_2}}\left(t_{s_2}\right) + N_3 \\ y_{r_1}\left(t_{s_2}\right) &= h_4^{r_1} x_{s_{r_1}}\left(t_{s_2}\right) + N_4 \end{aligned} \tag{16}$$

The decoded received SINR of UE_3 and UE_4 are:

$$\begin{aligned} SINR_{t_{s_2}}^{3-3} &= \frac{P_3^{t_{s_2}} P_{r_2} \left| h_3^{r_2} \right|^2}{\sigma^2} \\ SINR_{t_{s_2}}^{4-4} &= \frac{P_4^{t_{s_2}} P_{r_1} \left| h_4^{r_1} \right|^2}{\sigma^2} \end{aligned} \tag{17}$$

The minimum SINR of UE_3 and UE_4:

$$a_3^{t_{s2}} = \min\left(SINR_{t_{s1}}^{3-1}, SINR_{t_{s1}}^{3-2}, SINR_{t_{s1}}^{3-r_2}, SINR_{t_{s2}}^{3-3}\right) \quad (18)$$

$$a_4^{t_{s2}} = \min\left(SINR_{t_{s1}}^{4-1}, SINR_{t_{s1}}^{4-2}, SINR_{t_{s1}}^{4-r_1}, SINR_{t_{s1}}^{4-r_2}, SINR_{t_{s2}}^{4-4}\right) \quad (19)$$

The data rate of the UE_3 and UE_4 in time slot t_{s_2} is:

$$\begin{gathered} t_{s_2} log_2\left(1+a_3^{t_{s2}}\right) \\ t_{s_2} log_2\left(1+a_4^{t_{s2}}\right) \end{gathered} \quad (20)$$

The throughput achieved by UE_3 and UE_4 is:

$$\begin{gathered} Th_3 = n_{rb3} B\, log_2\left(1+a_3^{t_{s2}}\right) \\ Th_4 = n_{rb4} B\, log_2\left(1+a_4^{t_{s2}}\right) \end{gathered} \quad (21)$$

Time sub-slot t_{s3}

In this time slot, due to the mobility UE_2 changes its position and uses relay r_2 access link to connect with the BS. The BS transmits the superimposed signals of users 1, 2, 3, and 4 to the connected user equipment's UE_1, relays r_1 and r_2:

$$\begin{aligned} x_s\left(t_{s_3}\right) = {} & \sqrt{P_1^{t_{s3}} P_s}\, x_1\left(t_{s_3}\right) + \sqrt{P_2^{t_{s3}} P_s}\, x_2\left(t_{s_3}\right) \\ & + \sqrt{P_3^{t_{s3}} P_s}\, x_3\left(t_{s_3}\right) + \sqrt{P_4^{t_{s3}} P_s}\, x_4\left(t_{s_3}\right) \end{aligned} \quad (22)$$

where $x_1\left(t_{s_3}\right), x_2\left(t_{s_3}\right), x_3\left(t_{s_3}\right)$ and $x_4\left(t_{s_3}\right)$ are UE_1, UE_2, UE_3, and UE_4 data symbols with expected value equal to 1. The power allocation coefficients for UE_1, UE_2, UE_3, and UE_4 are $P_1^{t_{s3}}$, $P_2^{t_{s3}}$, $P_3^{t_{s3}}$ and $P_4^{t_{s3}}$, also $P_1^{t_{s3}} + P_2^{t_{s3}} + P_3^{t_{s3}} + P_4^{t_{s3}} = 1$ with $P_1^{t_{s3}} < P_2^{t_{s3}} < P_4^{t_{s3}} < P_3^{t_{s3}}$. The received signals at the UE_1, r_1 and r_2 are given as:

$$\begin{aligned} y_1\left(t_{s_3}\right) &= h_1^{BS} x_s\left(t_{s_3}\right) + N_1 \\ y_{r_1}\left(t_{s_3}\right) &= h_{r_1}^{BS} x_s\left(t_{s_3}\right) + N_{r_1} \\ y_{r_2}\left(t_{s_3}\right) &= h_{r_2}^{BS} x_s\left(t_{s_3}\right) + N_{r_2} \end{aligned} \quad (23)$$

where N_j, $j=1$, 2, and N_r denotes the AWGN at respective users or relays.

To retrieve its signal of interest, UE_1 uses SIC on the received superimposed signal to decode the signals meant for relays r_1 and r_2. In this case, the signal for UE_3 is decoded first and then UE_4 followed by UE_2. The signal to interference and noise ratio (SINR) are:

$$\begin{aligned} SINR_{t_{s3}}^{3-1} &= \frac{P_3^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s3}} P_s \left|h_1^{BS}\right|^2 + P_4^{t_{s3}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s3}}^{4-1} &= \frac{P_4^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s3}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s3}}^{2-1} &= \frac{P_2^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{P_1^{t_{s3}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \end{aligned} \quad (24)$$

After removing decoded SINR of UE_2 UE_3, and UE_4, the decoded SINR of UE_1 is:

$$SINR_{t_{s3}}^{1-1} = \frac{P_1^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{\sigma^2} \quad (25)$$

The data rate of the UE_1 in time slot t_{s_3} is:

$$t_{s_3} log_2\left(1+\frac{P_1^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{\sigma^2}\right) \quad (26)$$

The throughput achieved by UE_1 is:

$$Th_1 = n_{rb1} B\, log_2\left(1+\frac{P_1^{t_{s3}} P_s \left|h_1^{BS}\right|^2}{\sigma^2}\right) \quad (27)$$

The relay r_1 decodes the signal of UE_3 directly with interference from UE_1, UE_2, and UE_4 still present. The decoded SINR at relay r_1:

$$SINR_{t_{s3}}^{3-r_1} = \frac{P_3^{t_{s3}} P_s \left|h_{r_1}^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s3}} P_s \left|h_{r_1}^{BS}\right|^2 + P_4^{t_{s3}} P_s \left|h_{r_1}^{BS}\right|^2 + \sigma^2} \quad (28)$$

The relay r_2 decodes the signal of UE_3 and then removes the decoded signal. It then decodes the signal of UE_4 with interference from UE_1 and UE_2 still present. Finally, r_2 decodes the signal of UE_2 with interference from UE_1 still present. The decoded SINR at relay r_2 are:

$$\begin{aligned} SINR_{t_{s3}}^{3-r_2} &= \frac{P_3^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2 + P_4^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s3}}^{4-r_2} &= \frac{P_4^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2 + \sigma^2} \\ SINR_{t_{s3}}^{2-r_2} &= \frac{P_2^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2}{P_1^{t_{s3}} P_s \left|h_{r_2}^{BS}\right|^2 + \sigma^2} \end{aligned} \quad (29)$$

Time sub-slot t_{s4}

In this slot, the relays regenerate the new signals.

The signals generated by r_1 and r_2 are:

$$\begin{aligned} x_{s_{r_1}}\left(t_{s_4}\right) &= \sqrt{P_3^{t_{s4}} P_{r_1}}\, x_3\left(t_{s_3}\right) \\ x_{s_{r_2}}\left(t_{s_4}\right) &= \sqrt{P_2^{t_{s4}} P_{r_2}}\, x_2\left(t_{s_3}\right) + \sqrt{P_4^{t_{s4}} P_{r_2}}\, x_4\left(t_{s_3}\right) \end{aligned} \quad (30)$$

The power allocation coefficients for UE_2, UE_3, and UE_4 are $P_2^{t_{s4}}$, $P_3^{t_{s4}}$ and $P_4^{t_{s4}}$, also $P_2^{t_{s4}} + P_3^{t_{s4}} = 1$, $P_2^{t_{s4}} < P_4^{t_{s4}}$. The received signal at the UE_2 and UE_4 are given as:

$$\begin{aligned} y_2\left(t_{s_4}\right) &= h_2^{r_2} x_{s_{r_2}}\left(t_{s_4}\right) + N_2 \\ y_4\left(t_{s_4}\right) &= h_4^{r_2} x_{s_{r_2}}\left(t_{s_4}\right) + N_4 \end{aligned} \quad (31)$$

The received signal at the UE_3 is given as:

$$y_3\left(t_{s_4}\right) = h_3^{r_1} x_{s_{r_1}}\left(t_{s_4}\right) + N_3 \quad (32)$$

The UE_2 performs SIC for UE_4 signal and the decoded SINR is:

$$SINR_{t_{s_4}}^{4-2} = \frac{P_4^{t_{s4}} P_{r_2} \,|\, h_2^{r_2} \,|^2}{P_2^{t_{s4}} P_{r_2} \,|\, h_2^{r_2} \,|^2 + \sigma^2} \quad (33)$$

The decoded received SINR of UE_2 and UE_4 are:

$$\begin{aligned} SINR_{t_{s_4}}^{2-2} &= \frac{P_2^{t_{s4}} P_{r_2} \left|h_2^{r_2}\right|^2}{\sigma^2} \\ SINR_{t_{s_4}}^{4-4} &= \frac{P_4^{t_{s4}} P_{r_2} \,|\, h_4^{r_2} \,|^2}{P_2^{t_{s4}} P_{r_2} \,|\, h_4^{r_2} \,|^2 + \sigma^2} \end{aligned} \quad (34)$$

The minimum SINR for UE_2 and UE_4:

$$a_2^{t_{s4}} = \min\left(SINR_{t_{s_3}}^{2-1}, SINR_{t_{s_3}}^{2-r_2}, SINR_{t_{s_4}}^{2-2}\right) \quad (35)$$

$$a_4^{t_{s4}} = \min\left(SINR_{t_{s_3}}^{4-1}, SINR_{t_{s_3}}^{4-r_2}, SINR_{t_{s_4}}^{4-2}, SINR_{t_{s_4}}^{4-4}\right) \quad (36)$$

The decoded received SINR of UE_3 is:

$$SINR_{t_{s_4}}^{3-3} = \frac{P_3^{t_{s4}} P_{r_1} \,|\, h_3^{r_1} \,|^2}{\sigma^2} \quad (37)$$

The minimum SINR for UE_3:

$$a_3^{t_{s4}} = \min\left(SINR_{t_{s_3}}^{3-1}, SINR_{t_{s_3}}^{3-r_1}, SINR_{t_{s_3}}^{3-2r_2}, SINR_{t_{s_3}}^{r_1}, SINR_{t_{s_4}}^{3-3}\right) \quad (38)$$

The data rate of the UE_2, UE_3, and UE_4 in time slot t_{s_4} is:

$$t_{s_4} log_2\left(1 + a_x^{t_{s4}}\right) \quad (39)$$

The throughput achieved by the UE_2, UE_3, and UE_4 in time slot t_{s_4} is:

$$Th_x = n_{rbx} B\, log_2\left(1 + a_x^{t_{s4}}\right) \quad (40)$$

where $x = 2$, 3 and 4 and denote UE.

Outage probability

The outage probability (OP) is used to define the probability of occurrence of an outage event in the communication system. The outage event with respect to communication is the condition when a UE achieved rates (R) are less than the required rates. The outage probability of cell center use in this paper depends upon the direct link between UE and BS, while for a cell edge user, it depends upon both access and relay link.

OP for UE_1

Let $OE_{1,t_{s_1}}$ and $OE_{1,t_{s_3}}$ represents outage events for UE_1 in time sub-slots t_{s_1} and t_{s_3}. The OP in t_{s_1} and t_{s_3} are denoted by $P_{1,t_{s_1}} = P_r[OE_{1,t_{s_1}}]$ and $P_{1,t_{s_3}} = P_r[OE_{1,t_{s_3}}]$.

The UE_1 communication is interrupted, or an outage event may occur due to one of the possible events. If UE_1:

1. Cannot detect signals of UE_2, UE_3 and UE_4.
2. Throughput is not able to achieve the target rate R in time sub-slot.

OP for UE_1 in t_{s_1}

$$\begin{aligned} P_{1,t_{s_1}} &= P_r\left[OE_{1,t_{s_1}}\right] = P_r\left[OE_{r_{2-1}}^{t_{s1}} \cup OE_{r_{3-1}}^{t_{s1}} \cup OE_{r_{4-1}}^{t_{s1}} \cup OE_{r_{1-1}}^{t_{s1}}\right] \\ &= P_r\left[\left(R_{r_{2-1}}^{t_{s1}} < R\right) \cup \left(R_{r_{3-1}}^{t_{s1}} < R\right) \cup \left(R_{r_{4-1}}^{t_{s1}} < R\right) \cup \left(R_{r_{1-1}}^{t_{s1}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_{2-1}}^{t_{s1}}, R_{r_{3-1}}^{t_{s1}}, R_{r_{4-1}}^{t_{s1}}, R_{r_{1-1}}^{t_{s1}}) < R\right)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_{2-1}}^{t_{s1}}, R_{r_{3-1}}^{t_{s1}}, R_{r_{4-1}}^{t_{s1}}, R_{r_{1-1}}^{t_{s1}}) \geq R\right)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_{1-1}}^{t_{s1}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{2-1}}^{t_{s1}} \geq R\right)\right] \\ &\quad \left[1 - P_r\left(R_{r_{3-1}}^{t_{s1}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{4-1}}^{t_{s1}} \geq R\right)\right] \\ &= 1 - [1 - F_{R_{r_{2-1}}^{t_{s1}}}(R)][1 - F_{R_{r_{3-1}}^{t_{s1}}}(R)][1 - F_{R_{r_{4-1}}^{t_{s1}}}(R)][1 - F_{R_{r_{1-1}}^{t_{s1}}}(R)] \end{aligned} \quad (41)$$

$$F_{R_{r_{1-1}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1+SINR_{t_{s_1}}^{1-1}\right) < R\right)$$

$$F_{R_{r_{1-1}}^{t_{s_1}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_1}}-1)\sigma^2}{\lambda_1^{BS} P_1^{t_{s_1}} P_s}}, & (2^{R/t_{s_1}}-1) > 0 \\ 1, & else \end{cases} \quad (42)$$

$$F_{R_{r_{2-1}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1+SINR_{t_{s_1}}^{2-1}\right) < R\right)$$

$$F_{R_{r_{2-1}}^{t_{s_1}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_1}}-1)\sigma^2}{\lambda_1^{BS}[P_2^{t_{s_1}} - P_1^{t_{s_1}}(2^{R/t_{s_1}}-1)]P_s}}, & 0 < (2^{R/t_{s_1}}-1) < \frac{P_2^{t_{s_1}}}{P_1^{t_{s_1}}} \\ 1, & else \end{cases} \quad (43)$$

$$F_{R_{r_{3-1}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1+SINR_{t_{s_1}}^{3-1}\right) < R\right)$$

$$F_{R_{r_{3-1}}^{t_{s_1}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_1}}-1)\sigma^2}{\lambda_1^{BS}[P_3^{t_{s_1}} - \sum_{j=1}^{2} P_j^{t_{s_1}}(2^{R/t_{s_1}}-1)]P_s}}, & 0 < (2^{R/t_{s_1}}-1) < \frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_j^{t_{s_1}}} \\ 1, & else \end{cases} \quad (44)$$

$$F_{R_{r_{4-1}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1+SINR_{t_{s_1}}^{4-1}\right) < R\right)$$

$$F_{R_{r_{4-1}}^{t_{s_1}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_1}}-1)\sigma^2}{\lambda_1^{BS}[P_4^{t_{s_1}} - \sum_{j=1}^{3} P_j^{t_{s_1}}(2^{R/t_{s_1}}-1)]P_s}} & ,0 < (2^{R/t_{s_1}}-1) < \frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}}} \\ 1, & else \end{cases} \quad (45)$$

$$P_{1,t_{s_1}} = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_1}}-1)\sigma^2}{\lambda_1^{BS} P_s}\left[\frac{1}{P_1^{t_{s_1}}} + \frac{1}{[P_2^{t_{s_1}} - P_1^{t_{s_1}}(2^{R/t_{s_1}}-1)} + \frac{1}{[P_3^{t_{s_1}} - \sum_{j=1}^{2} P_j^{t_{s_1}}(2^{R/t_{s_1}}-1)]} + \frac{1}{[P_4^{t_{s_1}} - \sum_{j=1}^{3} P_j^{t_{s_1}}(2^{R/t_{s_1}}-1)]}\right]} & , (2^{R/t_{s_1}}-1) < \left\{\frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}}} \cup \frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_1^{t_{s_1}}} \cup \frac{P_2^{t_{s_1}}}{P_1^{t_{s_1}}}\right\} \\ 1, & else \end{cases} \quad (46)$$

OP for UE_1 in t_{s_3}

$$\begin{aligned} P_{1,t_{s_3}} &= P_r\left[OE_{1,t_{s_3}}\right] = P_r\left[OE_{r_{2-1}}^{t_{s_3}} \cup OE_{r_{3-1}}^{t_{s_3}} \cup OE_{r_{4-1}}^{t_{s_3}} \cup OE_{r_{1-1}}^{t_{s_3}}\right] \\ &= P_r\left[\left(R_{r_{2-1}}^{t_{s_3}} < R\right) \cup \left(R_{r_{3-1}}^{t_{s_3}} < R\right) \cup \left(R_{r_{4-1}}^{t_{s_3}} < R\right) \cup \left(R_{r_{1-1}}^{t_{s_3}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_{2-1}}^{t_{s_3}}, R_{r_{3-1}}^{t_{s_3}}, R_{r_{4-1}}^{t_{s_3}}, R_{r_{1-1}}^{t_{s_3}}) < R\right)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_{2-1}}^{t_{s_3}}, R_{r_{3-1}}^{t_{s_3}}, R_{r_{4-1}}^{t_{s_3}}, R_{r_{1-1}}^{t_{s_3}}) \geq R\right)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_{1-1}}^{t_{s_3}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{2-1}}^{t_{s_3}} \geq R\right)\right] \\ &\quad \left[1 - P_r\left(R_{r_{3-1}}^{t_{s_3}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{4-1}}^{t_{s_3}} \geq R\right)\right] \\ &= 1 - [1 - F_{R_{r_{2-1}}^{t_{s_3}}}(R)][1 - F_{R_{r_{3-1}}^{t_{s_3}}}(R)][1 - F_{R_{r_{4-1}}^{t_{s_3}}}(R)][1 - F_{R_{r_{1-1}}^{t_{s_3}}}(R)] \end{aligned} \quad (47)$$

$$F_{R_{r_{1-1}}^{t_{s_3}}}(R) = P_r\left(t_{s_3} log_2\left(1+SINR_{t_{s_3}}^{1-1}\right) < R\right)$$

$$F_{R_{r_{1-1}}^{t_{s_3}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_3}}-1)\sigma^2}{\lambda_1^{BS} P_1^{t_{s_3}} P_s}} & ,0 < (2^{R/t_{s_3}}-1) \\ 1, & else \end{cases} \quad (48)$$

$$F_{R_{r_{2-1}}^{t_{s_3}}}(R) = P_r\left(t_{s_3} log_2\left(1+SINR_{t_{s_3}}^{2-1}\right) < R\right)$$

$$F_{R_{r_{2-1}}^{t_{s_3}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_3}}-1)\sigma^2}{\lambda_1^{BS}[P_2^{t_{s_3}} - P_1^{t_{s_3}}(2^{R/t_{s_3}}-1)]P_s}}, & 0 < (2^{R/t_{s_3}}-1) < \frac{P_2^{t_{s_3}}}{P_1^{t_{s_3}}} \\ 1, & else \end{cases} \quad (49)$$

$$F_{R_{r_{3-1}}^{t_{s_3}}}(R) = P_r\left(t_{s_3} log_2\left(1+SINR_{t_{s_3}}^{3-1}\right) < R\right)$$

$$F_{R_{r_{3-1}}^{t_{s_3}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_3}}-1)\sigma^2}{\lambda_1^{BS}[P_3^{t_{s_3}} - (\sum_{j=1}^{2} P_j^{t_{s_3}} + P_4^{t_{s_3}})(2^{R/t_{s_3}}-1)]P_s}} & ,0 < (2^{R/t_{s_3}}-1) < \frac{P_3^{t_{s_3}}}{(\sum_{j=1}^{2} P_j^{t_{s_3}} + P_4^{t_{s_3}})} \\ 1, & else \end{cases} \quad (50)$$

$$F_{R_{r_{4-1}}^{t_{s_3}}}(R) = P_r\left(t_{s_3} log_2\left(1+SINR_{t_{s_3}}^{4-1}\right) < R\right)$$

$$F_{R_{r_{4-1}}^{t_{s_3}}}(R) = \begin{cases} 1-e^{-\frac{(2^{R/t_{s_3}}-1)\sigma^2}{\lambda_1^{BS}[P_4^{t_{s_3}} - \sum_{j=1}^{2} P_j^{t_{s_3}}(2^{R/t_{s_3}}-1)]P_s}}, & 0 < (2^{R/t_{s_3}}-1) < \frac{P_4^{t_{s_3}}}{\sum_{j=1}^{2} P_j^{t_{s_3}}} \\ 1, & else \end{cases} \quad (51)$$

$$P_{1,t_{s_3}} = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s3}}-1)\sigma^2}{\lambda_1^{BS} P_s}\left[\frac{1}{P_1^{t_{s3}}} + \frac{1}{[P_2^{t_{s3}} - P_1^{t_{s3}}(2^{R/t_{s3}}-1)]} + \frac{1}{[P_3^{t_{s3}} - (\sum_{j=1}^{2} P_j^{t_{s3}} + P_4^{t_{s3}})(2^{R/t_{s3}}-1)]} + \frac{1}{[P_4^{t_{s3}} - \sum_{j=1}^{2} P_j^{t_{s3}}(2^{R/t_{s3}}-1)]}\right]} \\ ,(2^{R/t_{s3}} - 1) < \left\{ \frac{P_4^{t_{s3}}}{\sum_{j=1}^{2} P_j^{t_{s3}}} \cup \frac{P_3^{t_{s3}}}{\sum_{j=1}^{2} P_j^{t_{s3}} + P_4^{t_{s3}}} \cup \frac{P_2^{t_{s3}}}{P_1^{t_{s3}}} \right\} \\ 1, \quad else \end{cases} \tag{52}$$

OP for UE_2

Let $OE_{2,t_{s_1}}$ and $OE_{2,t_{s_3}}$ represent outage events for UE_2 in time sub-slots t_{s_1} and t_{s_3}. The *OP* in t_{s_1} and t_{s_4} are denoted by $P_{2,t_{s_1}} = P_r[OE_{2,t_{s_1}}]$ and $P_{2,t_{s_3}} = P_r[OE_{1,t_{s_3}}]$.

OP for UE_2 *in* t_{s_1}

The UE_2 communication is interrupted, or an outage event may occur due to one of the possible events. If UE_2:

1. cannot detect signals of UE_3 and UE_4.
2. Throughput is not able to achieve target rate R in time sub-slot t_{s_1}.

$$\begin{aligned} P_{2,t_{s_1}} &= P_r\left[OE_{2,t_{s_1}}\right] = P_r\left[OE_{r_{3-2}}^{t_{s_1}} \cup OE_{r_{4-2}}^{t_{s_1}} \cup OE_{r_{2-2}}^{t_{s_1}}\right] \\ &= P_r\left[\left(R_{r_{3-2}}^{t_{s_1}} < R\right) \cup \left(R_{r_{4-2}}^{t_{s_1}} < R\right) \cup \left(R_{r_{2-2}}^{t_{s_1}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_{3-2}}^{t_{s_1}}, R_{r_{4-2}}^{t_{s_1}}, R_{r_{2-2}}^{t_{s_1}}) < R)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_{3-2}}^{t_{s_1}}, R_{r_{4-2}}^{t_{s_1}}, R_{r_{2-2}}^{t_{s_1}}) \geq R)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_{2-2}}^{t_{s_1}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{3-2}}^{t_{s_1}} \geq R\right)\right] \\ &\quad \left[1 - P_r\left(R_{r_{4-2}}^{t_{s_1}} \geq R\right)\right] \\ &= 1 - \left[1 - F_{R_{r_{2-2}}^{t_{s_1}}}(R)\right]\left[1 - F_{R_{r_{3-2}}^{t_{s_1}}}(R)\right]\left[1 - F_{R_{r_{4-2}}^{t_{s_1}}}(R)\right] \end{aligned} \tag{53}$$

$$F_{R_{r_{2-2}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1 + SINR_{t_{s_1}}^{2-2}\right) < R\right)$$

$$F_{R_{r_{2-2}}^{t_{s_1}}}(R) = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s1}}-1)\sigma^2}{\lambda_2^{BS}\left[P_2^{t_{s1}} - P_1^{t_{s1}}\left(2^{R/t_{s1}}-1\right)\right]P_s}}, \\ 0 < (2^{R/t_{s_1}} - 1) < \frac{P_2^{t_{s_1}}}{P_1^{t_{s_1}}} \\ 1, \quad else \end{cases} \tag{54}$$

$$F_{R_{r_{3-2}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1 + SINR_{t_{s_1}}^{3-2}\right) < R\right)$$

$$F_{R_{r_{3-2}}^{t_{s_1}}}(R) = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s1}}-1)\sigma^2}{\lambda_2^{BS}[P_3^{t_{s1}} - \sum_{j=1}^{2} P_j^{t_{s1}}(2^{R/t_{s1}}-1)]P_s}} \\ ,0 < (2^{R/t_{s_1}} - 1) < \frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_j^{t_{s_1}}} \\ 1, \quad else \end{cases} \tag{55}$$

$$F_{R_{r_{4-2}}^{t_{s_1}}}(R) = P_r\left(t_{s_1} log_2\left(1 + SINR_{t_{s_1}}^{4-2}\right) < R\right)$$

$$F_{R_{r_{4-2}}^{t_{s_1}}}(R) = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s1}}-1)\sigma^2}{\lambda_2^{BS}[P_4^{t_{s1}} - \sum_{j=1}^{3} P_j^{t_{s1}}(2^{R/t_{s1}}-1)]P_s}} \\ ,0 < (2^{R/t_{s_1}} - 1) < \frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}}} \\ 1, \quad else \end{cases} \tag{56}$$

$$P_{2,t_{s_1}} = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s1}}-1)\sigma^2}{\lambda_2^{BS} P_s}\left[\frac{1}{[P_2^{t_{s1}} - P_1^{t_{s1}}(2^{R/t_{s1}}-1)} + \frac{1}{[P_3^{t_{s1}} - \sum_{j=1}^{2} P_j^{t_{s1}}(2^{R/t_{s1}}-1)]} + \frac{1}{[P_4^{t_{s1}} - \sum_{j=1}^{3} P_j^{t_{s1}}(2^{R/t_{s1}}-1)]}\right]} \\ , (2^{R/t_{s_1}} - 1) < \left\{ \frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}}} \cup \frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_j^{t_{s_1}}} \cup \frac{P_2^{t_{s_1}}}{P_1^{t_{s_1}}} \right\} \\ 1, \quad else \end{cases} \tag{57}$$

OP for UE_2 *in* t_{s_3} *and* t_{s_4}

The UE_2 communication is interrupted, or an outage event may occur due to one of the possible events. If UE_2:

1. Cannot detect signals of UE_3 by relay and UE_4.
2. Throughput is not able to achieve the target rate R in time sub-slot t_{s_4} and if UE_2 signal is not detected by the relay in time sub-slot t_{s_3}.

Also, $t_{s_3} = t_{s_4}$.

$$\begin{aligned}
P_{2,t_{s_4}} &= P_r\left[OE_{2,t_{s_4}}\right] = P_r\left[OE_{r_{3-r_2}}^{t_{s4}} \cup OE_{r_{4-2}}^{t_{s4}} \cup OE_{r_2}^{t_{s3}} \cup OE_{r_2}^{t_{s4}}\right] \\
&= P_r\left[\left(R_{r_{3-r_2}}^{t_{s4}} < R\right) \cup \left(R_{r_{4-2}}^{t_{s4}} < R\right) \cup \left(R_{r_2}^{t_{s3}} < R\right) \cup \left(R_{r_2}^{t_{s4}} < R\right)\right] \\
&= P_r\left[min\left[(R_{r_{3-r_2}}^{t_{s4}}, R_{r_{4-2}}^{t_{s4}}, R_{r_2}^{t_{s3}}, R_{r_2}^{t_{s4}}) < R\right)\right]\right] \\
&= 1 - P_r\left[min\left[(R_{r_{3-r_2}}^{t_{s4}}, R_{r_{4-2}}^{t_{s4}}, R_{r_2}^{t_{s3}}, R_{r_2}^{t_{s4}}) \geq R\right)\right]\right] \\
&= 1 - \left[1 - P_r\left(R_{3-r_2}^{t_{s4}} \geq R\right)\right]\left[1 - P_r\left(R_{r_{4-2}}^{t_{s4}} \geq R\right)\right] \\
&\quad \left[1 - P_r\left(R_{r_2}^{t_{s3}} \geq R\right)\right]\left[1 - P_r\left(R_{r_2}^{t_{s4}} \geq R\right)\right] \\
&= 1 - \left[1 - F_{R_{r_2}^{t_{s4}}}(R)\right]\left[1 - F_{R_{r_{3-r_2}}^{t_{s4}}}(R)\right]\left[1 - F_{R_{r_{4-2}}^{t_{s4}}}(R)\right]\left[1 - F_{R_{r_2}^{t_{s3}}}(R)\right]
\end{aligned} \tag{58}$$

$$F_{R_{r_2}^{t_{s4}}}(R) = P_r\left(t_{s_4} log_2\left(1 + SINR_{t_{s_4}}^2\right) < R\right)$$

$$F_{R_{r_2}^{t_{s4}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s4}} - 1)\sigma^2 / \lambda_2^{r_2} P_2^{t_{s4}} P_s} & 0 < (2^{R/t_{s4}} - 1) \\ 1, & else \end{cases} \tag{59}$$

$$F_{R_{r_{3-r_2}}^{t_{s4}}}(R) = P_r\left(t_{s_4} log_2\left(1 + SINR_{t_{s_4}}^{3-r_2}\right) < R\right)$$

$$F_{R_{r_{3-r_2}}^{t_{s4}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s4}} - 1)\sigma^2 / \lambda_{r_2}^{BS}[P_3^{t_{s4}} - P_1^{t_{s4}}(2^{R/t_{s4}} - 1)]P_s}, \\ 0 < (2^{R/t_{s4}} - 1) < \dfrac{P_3^{t_{s4}}}{P_1^{t_{s4}}} \\ 1, & else \end{cases} \tag{60}$$

$$F_{R_{r_{4-2}}^{t_{s4}}}(R) = P_r\left(t_{s_4} log_2(1) < R\right)$$

$$F_{R_{r_{4-2}}^{t_{s4}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s4}} - 1)\sigma^2 / \lambda_2^{r_2}[P_4^{t_{s4}} - P_2^{t_{s4}}(2^{R/t_{s4}} - 1)]P_s}, & 0 < (2^{R/t_{s4}} - 1) < \dfrac{P_4^{t_{s4}}}{P_2^{t_{s4}}} \\ 1, & else \end{cases} \tag{61}$$

$$F_{R_{r_2}^{t_{s3}}}(R) = P_r\left(t_{s_3} log_2\left(1 + SINR_{t_{s3}}^{r_2}\right) < R\right)$$

$$F_{R_{r_2}^{t_{s3}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s3}} - 1)\sigma^2 / \lambda_{r_2}^{BS}[P_2^{t_{s3}} - P_1^{t_{s3}}(2^{R/t_{s3}} - 1)]P_s}, & 0 < (2^{R/t_{s3}} - 1) < \dfrac{P_4^{t_{s3}}}{P_2^{t_{s3}}} \\ 1, & else \end{cases} \tag{62}$$

$$P_{2,t_{s3}} = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s3}} - 1)\sigma^2}{P_s}\left[\frac{(2^{R/t_{s4}} - 1)}{\lambda_2^{r_2} P_2^{t_{s4}}} + \frac{(2^{R/t_{s4}} - 1)}{\lambda_{r_2}^{BS}[P_3^{t_{s4}} - P_1^{t_{s4}}(2^{R/t_{s4}} - 1)]} + \frac{(2^{R/t_{s3}} - 1)}{\lambda_{r_2}^{BS}[P_2^{t_{s3}} - P_1^{t_{s3}}(2^{R/t_{s3}} - 1)]}\right]} \\ ,(2^{R/t_{s_4}} - 1) < \dfrac{P_3^{t_{s_4}}}{P_1^{t_{s_4}}} \cup (2^{R/t_{s_3}} - 1) < \dfrac{P_2^{t_{s_3}}}{P_1^{t_{s_3}}} \\ 1, & else \end{cases} \tag{63}$$

OP for UE_3

OP for UE_3 in t_{s_1} and t_{s_2}

The UE_3 communication is interrupted, or an outage event may occur due to one of the possible events. If UE_3:

1. Signal is not detected at the relay r_2 in time sub-slot t_{s_1}.
2. Throughput is not able to achieve the target rate R on access link in time sub-slot t_{s_2}.

Also, $t_{s_1} = t_{s_2}$.

$$\begin{aligned}
P_{3,t_{s_2}} &= P_r\left[OE_{3,t_{s_2}}\right] = P_r\left[OE_{r_2}^{t_{s_1}} \cup OE_{r_3}^{t_{s_2}}\right] \\
&= P_r\left[\left(R_{r_2}^{t_{s_1}} < R\right) \cup \left(R_{r_3}^{t_{s_2}} < R\right)\right] \\
&= P_r\left[min\left[(R_{r_2}^{t_{s_1}}, R_{r_3}^{t_{s_2}}) < R\right)\right]\right] \\
&= 1 - P_r\left[min\left[(R_{r_2}^{t_{s_1}}, R_{r_3}^{t_{s_2}}) \geq R\right)\right]\right] \\
&= 1 - \left[1 - P_r\left(R_{r_2}^{t_{s_1}} \geq R\right)\right]\left[1 - P_r\left(R_{r_3}^{t_{s_2}} \geq R\right)\right] \\
&= 1 - \left[1 - F_{R_{r_2}^{t_{s1}}}(R)\right]\left[1 - F_{R_{r_3}^{t_{s2}}}(R)\right]
\end{aligned} \tag{64}$$

$$F_{R_{r_2}^{t_{s1}}}(R) = P_r\left(t_{s_1} log_2\left(1 + SINR_{t_{s_1}}^{r_2}\right) < R\right)$$

$$F_{R_{r_2}^{t_{s1}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s1}} - 1)\sigma^2 / \lambda_{r_2}^{BS}\left[P_3^{t_{s1}} - \sum_{j=1}^{2} P_j^{t_{s1}}(2^{R/t_{s1}} - 1)\right]P_s} \\ ,0 < (2^{R/t_{s_1}} - 1) < \dfrac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_j^{t_{s_1}}(2^{R/t_{s_1}} - 1)} \\ 1, & else \end{cases} \tag{65}$$

$$F_{R_{r_3}^{t_{s2}}}(R) = P_r\left(t_{s_2} log_2\left(1 + SINR_{t_{s_2}}^3\right) < R\right)$$

$$F_{R_{r_3}^{t_{s2}}}(R) = \begin{cases} 1 - e^{-(2^{R/t_{s2}} - 1)\sigma^2 / \lambda_3^{r_2} P_3^{t_{s2}} P_s}, & 0 < (2^{R/t_{s_2}} - 1) \\ 1, & else \end{cases} \tag{66}$$

$$P_{3,t_{s_1}} = \begin{cases} 1 - e^{-\frac{\sigma^2}{P_s}\left[\frac{(2^{R/t_{s1}} - 1)}{\lambda_{r_2}^{BS}[P_3^{t_{s1}} - \sum_{j=1}^{2} P_j^{t_{s1}}(2^{R/t_{s1}} - 1)]} + \frac{(2^{R/t_{s2}} - 1)}{\lambda_3^{r_2} P_3^{t_{s2}}}\right]} \\ , (2^{R/t_{s_1}} - 1) < \dfrac{P_3^{t_{s_1}}}{\sum_{j=1}^{2} P_j^{t_{s_1}}} \cup 0 < (2^{R/t_{s_2}} - 1) \\ 1, & else \end{cases} \tag{67}$$

OP for UE_3 in t_{s_3} and t_{s_4}

The UE_3 communication is interrupted in these time subs lots due to an outage event and may occur due to one of the possible events. If UE_3:

1. Signal is not detected at the relay r_1 in time sub-slot t_{s_3}.
2. Throughput is not able to achieve target rate R on access link in time sub-slot t_{s_4}.

Also, $t_{s_3} = t_{s_4}$.

$$\begin{aligned} P_{3,t_{s_4}} &= P_r\left[OE_{3,t_{s_4}}\right] = P_r\left[OE_{r_1}^{t_{s3}} \cup OE_{r_3}^{t_{s4}}\right] \\ &= P_r\left[\left(R_{r_1}^{t_{s3}} < R\right) \cup \left(R_{r_3}^{t_{s4}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_1}^{t_{s3}}, R_{r_3}^{t_{s4}}) < R)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_1}^{t_{s3}}, R_{r_3}^{t_{s4}}) \geq R)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_1}^{t_{s3}}, R_{r_3}^{t_{s4}}) \geq R)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_1}^{t_{s3}} \geq R\right)\right]\left[1 - P_r\left(R_{r_3}^{t_{s4}} \geq R\right)\right] \\ &= 1 - [1 - F_{R_{r_1}^{t_{s3}}}(R)][1 - F_{R_{r_3}^{t_{s4}}}(R)] \end{aligned} \tag{68}$$

$$F_{R_{r_1}^{t_{s3}}}(R) = P_r\left(t_{s_3} log_2\left(1 + SINR_{t_{s_3}}^{r_1}\right) < R\right)$$

$$F_{R_{r_1}^{t_{s3}}}(R) = \left\{\begin{matrix} 1 - e^{-(2^{R/t_{s3}} - 1)\sigma^2 / \lambda_{r_1}^{BS}[P_3^{t_{s3}} - \left(\sum_{j=1}^{2} P_j^{t_{s3}} + P_4^{t_{s3}})\right]P_s} \\ , 0 < (2^{R/t_{s3}} - 1) < \dfrac{P_3^{t_{s_3}}}{(\sum_{j=1}^{2} P_j^{t_{s_3}} + P_4^{t_{s_3}})} \\ 1, \quad else \end{matrix}\right\} \tag{69}$$

$$F_{R_{r_3}^{t_{s4}}}(R) = P_r\left(t_{s_4} log_2\left(1 + SINR_{t_{s_4}}^{r_3}\right) < R\right)$$

$$F_{R_{r_3}^{t_{s4}}}(R) = \left\{\begin{matrix} 1 - e^{-(2^{R/t_{s4}} - 1)\sigma^2 / \lambda_3^{r_1} P_3^{t_{s4}} P_s}, 0 < (2^{R/t_{s4}} - 1) \\ 1, \quad else \end{matrix}\right\} \tag{70}$$

$$P_{3,t_{s_3}} = \left\{\begin{matrix} 1 - e^{-\frac{\sigma^2}{P_s}\left[\frac{(2^{R/t_{s3}} - 1)}{\lambda_{r_1}^{BS}[P_3^{t_{s3}} - \sum_{j=1}^{2} P_j^{t_{s3}} (2^{R/t_{s3}} - 1)]} + \frac{(2^{R/t_{s4}} - 1)}{\lambda_3^{r_1} P_3^{t_{s4}}}\right]} \\ , (2^{R/t_{s_3}} - 1) < \dfrac{P_3^{t_{s_3}}}{\sum_{j=1}^{2} P_j^{t_{s_3}}} \cup 0 < (2^{R/t_{s_4}} - 1) \\ 1, \quad else \end{matrix}\right\} \tag{71}$$

OP for *UE_4*

OP for UE_4 in t_{s_1} and t_{s_2}

The UE_4 communication is interrupted, or an outage event may occur due to one of the possible events. If UE_4:

1. Signal is not detected at the relay r_1 in time sub-slot t_{s_1}.
2. Throughput is not able to achieve the target rate R on access link in time sub-slot t_{s_2}.

Also, $t_{s_1} = t_{s_2}$

$$\begin{aligned} P_{4,t_{s_2}} &= P_r\left[OE_{4,t_{s_2}}\right] = P_r\left[OE_{r_1}^{t_{s1}} \cup OE_{r_4}^{t_{s2}}\right] \\ &= P_r\left[\left(R_{r_1}^{t_{s1}} < R\right) \cup \left(R_{r_4}^{t_{s2}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_1}^{t_{s1}}, R_{r_4}^{t_{s2}}) < R)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_1}^{t_{s1}}, R_{r_4}^{t_{s2}}) \geq R)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_1}^{t_{s1}} \geq R\right)\right]\left[1 - P_r\left(R_{r_4}^{t_{s2}} \geq R\right)\right] \\ &= 1 - \left[1 - F_{R_{r_1}^{t_{s1}}}(R)\right]\left[1 - F_{R_{r_4}^{t_{s2}}}(R)\right] \end{aligned} \tag{72}$$

$$F_{R_{r_1}^{t_{s1}}}(R) = P_r\left(t_{s_1} log_2\left(1 + SINR_{t_{s_1}}^{r_1}\right) < R\right)$$

$$F_{R_{r_1}^{t_{s1}}}(R) = \left\{\begin{matrix} 1 - e^{-(2^{R/t_{s1}} - 1)\sigma^2 / \lambda_{r_1}^{BS}\left[P_4^{t_{s1}} - \sum_{j=1}^{3} P_j^{t_{s1}} (2^{R/t_{s1}} - 1\right)]P_s} \\ , 0 < (2^{R/t_{s_1}} - 1) < \dfrac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}} (2^{R/t_{s_1}} - 1)} \\ 1, \quad else \end{matrix}\right\} \tag{73}$$

$$F_{R_{r_4}^{t_{s2}}}(R) = P_r\left(t_{s_2} log_2\left(1 + SINR_{t_{s_2}}^{4}\right) < R\right)$$

$$F_{R_{r_4}^{t_{s2}}}(R) = \left\{\begin{matrix} 1 - e^{-(2^{R/t_{s2}} - 1)\sigma^2 / \lambda_4^{r_1} P_4^{t_{s2}} P_s}, 0 < (2^{R/t_{s2}} - 1) \\ 1, \quad else \end{matrix}\right\} \tag{74}$$

$$P_{4,t_{s_1}} = \left\{\begin{matrix} 1 - e^{-\frac{\sigma^2}{P_s}\left[\frac{(2^{R/t_{s1}} - 1)}{\lambda_{r_1}^{BS}[P_4^{t_{s1}} - \sum_{j=1}^{3} P_j^{t_{s1}} (2^{R/t_{s1}} - 1)]P_s} + \frac{(2^{R/t_{s2}} - 1)}{\lambda_4^{r_1} P_4^{t_{s2}}}\right]} \\ , (2^{R/t_{s_1}} - 1) < \dfrac{P_4^{t_{s_1}}}{\sum_{j=1}^{3} P_j^{t_{s_1}}} \cup 0 < (2^{R/t_{s_2}} - 1) \\ 1, \quad else \end{matrix}\right\} \tag{75}$$

OP for UE_4 in t_{s_3} and t_{s_4}

The UE_4 communication is interrupted in the time subs lots due to an outage event and may occur due to one of the possible events. If UE_4:

1. Signal is not detected at the relay r_2 in time sub-slot t_{s_3}.
2. Throughput is not able to achieve target rate R on access link in time sub-slot t_{s_4}.

Also, $t_{s_3} = t_{s_4}$.

$$\begin{aligned} P_{4,t_{s4}} &= P_r\left[OE_{4,t_{s4}}\right] = P_r\left[OE_{r_2}^{t_{s3}} \cup OE_{r_4}^{t_{s4}}\right] \\ &= P_r\left[\left(R_{r_2}^{t_{s3}} < R\right) \cup \left(R_{r_4}^{t_{s4}} < R\right)\right] \\ &= P_r\left[min\left[(R_{r_2}^{t_{s3}}, R_{r_4}^{t_{s4}}) < R)\right]\right] \\ &= 1 - P_r\left[min\left[(R_{r_2}^{t_{s3}}, R_{r_4}^{t_{s4}}) \geq R)\right]\right] \\ &= 1 - \left[1 - P_r\left(R_{r_2}^{t_{s3}} \geq R\right)\right]\left[1 - P_r\left(R_{r_4}^{t_{s4}} \geq R\right)\right] \\ &= 1 - [1 - F_{R_{r_2}^{t_{s3}}}(R)][1 - F_{R_{r_4}^{t_{s4}}}(R)] \end{aligned} \tag{76}$$

$$F_{R_{r_2}^{t_{s3}}}(R) = P_r\left(t_{s_3} log_2\left(1 + SINR_{t_{s3}}^{r_2}\right) < R\right)$$

$$F_{R_{r_2}^{t_{s3}}}(R) = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s3}} - 1)\sigma^2}{\lambda_{r_2}^{BS}\left[P_4^{t_{s3}} - \sum_{j=1}^{2} P_j^{t_{s3}}(2^{R/t_{s3}} - 1)\right]P_s}}, 0 < (2^{R/t_{s3}} - 1) < \dfrac{P_4^{t_{s3}}}{\sum_{j=1}^{2} P_j^{t_{s3}}(2^{R/t_{s3}} - 1)} \\ 1, \quad else \end{cases} \tag{77}$$

$$F_{R_{r_4}^{t_{s4}}}(R) = P_r\left(t_{s_4} log_2\left(1 + SINR_{t_{s4}}^{r_4}\right) < R\right)$$

$$F_{R_{r_4}^{t_{s4}}}(R) = \begin{cases} 1 - e^{-\frac{(2^{R/t_{s4}} - 1)\sigma^2}{\lambda_4^{r_2} P_4^{t_{s4}} P_s}}, 0 < (2^{R/t_{s4}} - 1) \\ 1, \quad else \end{cases} \tag{78}$$

$$P_{4,t_{s_3}} = \begin{cases} 1 - e^{-\frac{\sigma^2}{P_s}\left[\frac{(2^{R/t_{s3}} - 1)}{\lambda_{r_2}^{BS}[P_4^{t_{s3}} - \sum_{j=1}^{2} P_j^{t_{s3}}(2^{R/t_{s3}} - 1)]} + \frac{(2^{R/t_{s4}} - 1)}{\lambda_4^{r_2} P_3^{t_{s4}}}\right]}, (2^{R/t_{s3}} - 1) < \dfrac{P_4^{t_{s3}}}{\sum_{j=1}^{2} P_j^{t_{s3}}} \cup 0 < (2^{R/t_{s4}} - 1) \\ 1, \quad else \end{cases} \tag{79}$$

The total system outage event is the condition when no user can achieve detection, the total outage probability is:

$$P_T = 1 - \prod_{n=1}^{4}(1 - (P_{n,t_{s_1}})\prod_{n=1}^{4}(1 - P_{n,t_{s_3}}) \tag{80}$$

Ergodic rate

The ergodic rate of the UE in the sub-slot t is given by:

$$E_{UE}^{t} = \frac{t}{log2}\int_0^{\infty} log(1+y) f_Y(y) dy \tag{81}$$

$$E_{UE}^{t} = \frac{t}{log2}\int_0^{\infty} \frac{1 - F_Y(y)}{(1+y)} dy \tag{82}$$

where $f_y(y)$ and $F_y(y)$ denotes cumulative distribution function (CDF) and probability density function (PDF).

$$E_1^{t_{s_1}} = \frac{t_{s_1}}{log2}\int_0^{\infty} \frac{1 - F_{SINR_{t_{s_1}}^1}(y)}{(1+y)} dy \tag{83}$$

SINR is denoted by y, where $F_{SINR_{t_{s_1}}^1}$ denotes CDF of $SINR_{t_{s_1}}^1$, i.e:

$$F_{SINR_{t_{s_1}}^1}(y) = P\left(\frac{P_1^{t_{s_1}} P_s \left|h_1^{BS}\right|^2}{\sigma^2} \leq y\right) = P(|\, h_1^{BS}\,|^2 \leq \frac{y\sigma^2}{P_1^{t_{s_1}} P_s}) \tag{84}$$

Since the channel is complex Gaussian distribution and $|\, h_1^{BS}\,|^2$ is exponentially distributed with $1/\lambda_1^{BS}$:

$$F_{SINR_{t_{s_1}}^1}(y) = 1 - e^{-y\sigma^2 / P_1^{t_{s_1}} P_s \lambda_1^{BS}} \tag{85}$$

The ergodic rate of UE_1 in time sub-slot t_{s_1} is:

$$E_1^{t_{s_1}} = -\frac{t_{s_1}}{log2} Ei\left(\frac{-1}{P_1^{t_{s_1}} P_s \lambda_1^{BS}}\right) e^{1 / P_1^{t_{s_1}} P_s \lambda_1^{BS}} \tag{86}$$

In t_{s_3}, if UE_1 can perform SIC for UE_2, UE_3, and UE_4 successfully then:

$$F_{SINR_{t_{s3}}^1}(y) = 1 - e^{-y\sigma^2 / P_1^{t_{s3}} P_s \lambda_1^{BS}} \tag{87}$$

The ergodic rate of UE_3 in time sub-slot t_{s_3} is:

$$E_1^{t_{s_3}} = -\frac{t_{s_3}}{log2} Ei\left(\frac{-1}{P_1^{t_{s_3}} P_s \lambda_1^{BS}}\right) e^{1/P_1^{t_{s_3}} P_s \lambda_1^{BS}} \quad E_1 = E_1^{t_{s_1}} + E_1^{t_{s_3}} \tag{88}$$

UE_2 Ergodic rate: time slot t_{s_1}:

$$\begin{aligned} F^2_{SINR_{t_{s_1}}}(y) &= P\left(a_2^{t_{s_1}} \le y\right) = 1 - P\left(a_2^{t_{s_1}} > y\right) \\ &= 1 - [P\left(SINR^2_{t_{s_1}} > y\right) P\left(SINR^{2-1}_{t_{s_1}} > y\right) \\ &= \left[\left(1 - F_{SINR^2_{t_{s_1}}}(y)\right)\left(1 - F_{SINR^{2-1}_{t_{s_1}}}(y)\right)\right] \end{aligned} \tag{89}$$

$$\begin{aligned} F_{SINR^{2-1}_{t_{s_1}}}(y) &= P\left(\frac{P_2^{t_{s_1}} P_s |h_1^{BS}|^2}{P_1^{t_{s_1}} P_s |h_1^{BS}|^2 + \sigma^2} \le y\right) \\ &= P\left(\left|h_1^{BS}\right|^2 \le \frac{y\sigma^2}{\left(P_2^{t_{s_1}} - yP_1^{t_{s_1}}\right)P_s}\right) \\ F_{SINR^{2-1}_{t_{s_1}}}(y) &= 1 - e^{-y\sigma^2 / \left(P_2^{t_{s_1}} - yP_1^{t_{s_1}}\right)P_s \lambda_1^{BS}} \end{aligned} \tag{90}$$

$$\begin{aligned} F_{SINR^2_{t_{s_1}}}(y) &= P\left(\frac{P_2^{t_{s_1}} P_s \left|h_2^{BS}\right|^2}{P_1^{t_{s_1}} P_s \left|h_2^{BS}\right|^2 + \sigma^2} \le y\right) \\ &= P\left(\left|h_2^{BS}\right|^2 \le \frac{y\sigma^2}{\left(P_2^{t_{s_1}} - yP_1^{t_{s_1}}\right)P_s}\right) \\ F_{SINR^2_{t_{s_1}}}(y) &= 1 - e^{-y\sigma^2 / \left(P_2^{t_{s_1}} - yP_1^{t_{s_1}}\right)P_s \lambda_2^{BS}} \end{aligned} \tag{91}$$

$$F^2_{SINR_{t_{s_1}}}(y) = \begin{cases} 1 - e^{\frac{-y\sigma^2}{(P_2^{t_{s_1}} - yP_1^{t_{s_1}})P_s}\left(\frac{1}{\lambda_1^{BS}} + \frac{1}{\lambda_2^{BS}}\right)} & , y > 0 \\ 1 & , y \le 0 \end{cases} \tag{92}$$

The closed-form expression of $F^2_{SINR_{t_{s_1}}}(y)$ can be obtained by considering a high SINR situation for which transmission power of BS is infinite, i.e., $P_s \to \infty$, which changes:

$$F^2_{SINR_{t_{s_1}}}(y) = \begin{cases} 1 - e^{-\frac{y\sigma^2}{P_s}\left(\frac{1}{\lambda_1^{BS}} + \frac{1}{\lambda_2^{BS}}\right)} & , 0 < y < \frac{P_2^{t_{s_1}}}{P_1^{t_{s_1}}} \\ 1 & , else \end{cases} \tag{93}$$

The ergodic rate of UE_2 in time sub-slot t_{s_1} is:

$$E_2^{t_{s_1}} = -\frac{t_{s_1}}{log2} Ei\left(\frac{-1}{P_1^{t_{s_1}} P_s \lambda_1^{BS}}\right) e^{1/P_1^{t_{s_1}} P_s \lambda_1^{BS}} \tag{94}$$

time slot $t_{s_{3-4}}$

$$P_2^{t_{s_3}} + P_4^{t_{s_3}} = 1 \tag{95}$$

$$\begin{aligned} F^2_{SINR_{t_{s_4}}}(y) &= P\left(a_2^{t_{s_4}} \le y\right) = 1 - P\left(a_2^{t_{s_4}} > y\right) = 1 - P\left(a_2^{t_{s_4}} > y\right) \\ &= 1 - [P\left(SINR^{2-1}_{t_{s_3}} > y\right) P\left(SINR^{r_2-2}_{t_{s_3}} > y\right) P\left(SINR^2_{t_{s_4}}\right) \\ &= \left[1 - \left(1 - F_{SINR^{2-1}_{t_{s_3}}}(y)\right)\left(1 - F_{SINR^{r_2-2}_{t_{s_3}}}(y)\right)\left(1 - F_{SINR^2_{t_{s_4}}}(y)\right)\right] \end{aligned} \tag{96}$$

$$F_{SINR^{2-1}_{t_{s_3}}}(y) = P\left(\frac{P_2^{t_{s_3}} P_s \left|h_1^{BS}\right|^2}{P_1^{t_{s_3}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \le y\right) \tag{97}$$

$$F_{SINR^2_{t_{s_1}}}(y) = 1 - e^{-y\sigma^2 / (P_2^{t_{s_3}} - yP_1^{t_{s_3}})P_s \lambda_1^{BS}} \tag{98}$$

$$F_{SINR^{r_2-2}_{t_{s_3}}}(y) = P\left(\frac{P_2^{t_{s_3}} P_s \left|h_{r_2}^{BS}\right|^2}{P_1^{t_{s_3}} P_s \left|h_{r_2}^{BS}\right|^2 + \sigma^2} \le y\right) = 1 - e^{-y\sigma^2 / (P_2^{t_{s_3}} - yP_1^{t_{s_3}})P_s \lambda_{r_2}^{BS}} \tag{99}$$

$$F_{SINR^2_{t_{s_4}}}(y) = P\left(\frac{P_2^{t_{s_4}} P_{r_2} \left|h_2^{r_2}\right|^2}{\sigma^2} \le y\right) = 1 - e^{-y\sigma^2 / P_2^{t_{s_4}} P_{r_2} \lambda_2^{r_2}} \tag{100}$$

$$F^2_{SINR_{t_{s_{3-4}}}}(y) = \begin{cases} 1 - e^{-y\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}} + \frac{1}{P_s\lambda_{r_2}^{BS}} + \frac{1}{P_{r_2}\lambda_2^{r_2}}\right)} & , 0 < y < \frac{P_2^{t_{s_3}}}{P_1^{t_{s_3}}} \\ 1 & , else \end{cases} \tag{101}$$

For the closed-form with $P_s \to \infty, P_{r_1} \to \infty$ and $P_{r_2} \to \infty$. Also, assuming, $\frac{P_2^{t_{s_3}}}{P_1^{t_{s_3}}} = b$ and $\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}} + \frac{1}{P_s\lambda_{r_2}^{BS}} + \frac{1}{P_{r_2}\lambda_2^{r_2}}\right) = a$.

The ergodic rate of UE_2 in time sub-slot $t_{s_{3-4}}$ is:

$$E_2^{t_{s_{3-4}}} = -\frac{t_{s_3}}{log2}[(Ei(a) - (Ei(ab + a)]e^a \tag{102}$$

UE_3 Ergodic rate: time slot t_{s_2}.

$$\begin{aligned} F^3_{SINR_{t_{s_2}}}(y) &= P\left(a_3^{t_{s_2}} \le y\right) = 1 - P\left(a_3^{t_{s_2}} > y\right) \\ &= 1 - P(SINR^{3-1}_{t_{s_1}} > y)P(SINR^{3-2}_{t_{s_1}} > y)P(SINR^{r_2}_{t_{s_1}} >)P(SINR^3_{t_{s_2}} > y) \\ &= \left[1 - \left(1 - F_{SINR^{3-1}_{t_{s_1}}}(y)\right)\left(1 - F_{SINR^{3-2}_{t_{s_1}}}(y)\right)\left(1 - F_{SINR^{r_2}_{t_{s_1}}}(y)\right)\left(1 - F_{SINR^3_{t_{s_2}}}(y)\right)\right] \end{aligned} \tag{103}$$

$$F_{SINR^{3-1}_{t_{s_1}}}(y) = P\left(\frac{P_3^{t_{s_1}} P_s \left|h_1^{BS}\right|^2}{\sum_{j=1}^{2} P_j^{t_{s_1}} P_s \left|h_1^{BS}\right|^2 + \sigma^2} \le y\right) = 1 - e^{-y\sigma^2 / (P_3^{t_{s_1}} - \sum_{j=1}^{2} P_j^{t_{s_1}})P_s \lambda_1^{BS}} \tag{104}$$

$$F_{SINR_{t_{s_1}}^{3-2}}(y)=P\left(\frac{P_3^{t_{s_1}}P_s\left|h_2^{BS}\right|^2}{\sum_{j=1}^{2}P_j^{t_{s_1}}P_s\left|h_2^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_3^{t_{s1}}-\sum_{j=1}^{2}P_j^{t_{s1}})P_s\lambda_2^{BS}} \quad (105)$$

$$F_{SINR_{t_{s_1}}^{r_2}}(y)=P\left(\frac{P_3^{t_{s_1}}P_s\left|h_{r_2}^{BS}\right|^2}{\sum_{j=1}^{2}P_j^{t_{s_1}}P_s\left|h_{r_2}^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_3^{t_{s1}}-\sum_{j=1}^{2}P_j^{t_{s1}})P_s\lambda_{r_2}^{BS}} \quad (106)$$

$$F_{SINR_{t_{s_2}}^{3}}(y)=P\left(\frac{P_3^{t_{s_2}}P_{r_2}\left|h_3^{r_2}\right|^2}{\sigma^2}\le y\right)=1-e^{-y\sigma^2/P_3^{t_{s2}}P_s\lambda_3^{r2}} \quad (107)$$

$$F_{SINR_{t_{s_{1-2}}}}^{3}(y)=\begin{cases}1-e^{-y\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_2^{BS}}+\frac{1}{P_s\lambda_{r_2}^{BS}}+\frac{1}{P_{r_1}\lambda_3^{r_2}}\right)}, 0<y<\frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2}P_j^{t_{s_1}}}\\ 1 \quad , else\end{cases} \quad (108)$$

For the closed-form with $P_s\to\infty, P_{r_1}\to\infty$ and $P_{r_2}\to\infty$. Also, assuming, $\frac{P_3^{t_{s_1}}}{\sum_{j=1}^{2}P_j^{t_{s_1}}}=b$ and $\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_2^{BS}}+\frac{1}{P_s\lambda_{r_2}^{BS}}+\frac{1}{P_{r_1}\lambda_3^{r_2}}\right)=a$. The ergodic rate of UE_3 in time sub-slot $t_{s_{1-2}}$ is:

$$E_3^{t_{s_{1-2}}}=-\frac{t_{s_1}}{log2}[Ei(a)-Ei(ab+a)]e^a \quad (109)$$

UE_3 Ergodic rate: time slot $t_{s_{3-4}}$.

$$\begin{aligned}F_{SINR_{t_{s_{3-4}}}}^{3}(y)&=P\left(a_3^{t_{s_4}}\le y\right)=1-P\left(a_3^{t_{s_4}}>y\right)\\&=1-P(SINR_{t_{s_3}}^{3-1}>y)P(SINR_{t_{s_3}}^{3-r_2}>y)P(SINR_{t_{s_3}}^{r_1}>)P(SINR_{t_{s_4}}^{3}>y)\\&=\left[1-\left(1-F_{SINR_{t_{s_3}}^{3-1}}(y)\right)\left(1-F_{SINR_{t_{s_3}}^{3-r_2}}(y)\right)\left(1-F_{SINR_{t_{s_3}}^{r_1}}(y)\right)\left(1-F_{SINR_{t_{s_4}}^{3}}(y)\right)\right]\end{aligned} \quad (110)$$

$$F_{SINR_{t_{s_3}}^{3-1}}(y)=P\left(\frac{P_3^{t_{s_3}}P_s\left|h_1^{BS}\right|^2}{\sum_{j=1}^{2}P_j^{t_{s_3}}P_s\left|h_1^{BS}\right|^2+P_4^{t_{s_3}}P_s\left|h_1^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_3^{t_{s3}}-(\sum_{j=1}^{2}P_j^{t_{s3}}+P_4^{t_{s3}})y)P_s\lambda_1^{BS}} \quad (111)$$

$$F_{SINR_{t_{s_3}}^{3-r_2}}(y)=P\left(\frac{P_3^{t_{s_3}}\left|h_{r_2}^{BS}\right|^2}{P_1^{t_{s_3}}P_s\left|h_{r_2}^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_3^{t_{s3}}-P_4^{t_{s3}}y)P_s\lambda_{r_2}^{BS}} \quad (112)$$

$$F_{SINR_{t_{s_3}}^{r_1}}(y)=P\left(\frac{P_3^{t_{s_3}}P_s\left|h_{r_1}^{BS}\right|^2}{\sum_{j=1}^{2}P_j^{t_{s_3}}P_s\left|h_{r_1}^{BS}\right|^2+P_4^{t_{s_3}}P_s\left|h_{r_1}^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_3^{t_{s3}}-(\sum_{j=1}^{2}P_j^{t_{s3}}+P_4^{t_{s3}})y)P_s\lambda_{r_1}^{BS}} \quad (113)$$

$$F_{SINR_{t_{s_4}}^{3}}(y)=P\left(\frac{P_3^{t_{s_4}}P_{r_1}\left|h_3^{r_1}\right|^2}{\sigma^2}\le y\right)=1-e^{-y\sigma^2/P_3^{t_{s4}}P_{r_1}\lambda_3^{r_1}} \quad (114)$$

$$F_{SINR_{t_{s_{3-4}}}}^{3}(y)=\begin{cases}1-e^{-y\sigma^2\left(\frac{1}{(P_3^{t_{s3}}-(\sum_{j=1}^{2}P_j^{t_{s3}}+P_4^{t_{s3}})y)P_s\lambda_1^{BS}}+\frac{1}{(P_3^{t_{s3}}-P_4^{t_{s3}}y)P_s\lambda_{r_2}^{BS}}+\frac{1}{(P_3^{t_{s3}}-(\sum_{j=1}^{2}P_j^{t_{s3}}+P_4^{t_{s3}})y)P_s\lambda_{r_1}^{BS}}+\frac{1}{P_3^{t_{s4}}P_{r_1}\lambda_3^{r_1}}\right)}\\ , 0<y<\frac{P_3^{t_{s_3}}}{\sum_{j=1}^{2}P_j^{t_{s_3}}+P_4^{t_{s_3}}}, 0<y<\frac{P_3^{t_{s_3}}}{P_4^{t_{s_3}}}\\ 1 \quad , else\end{cases} \quad (115)$$

For the closed-form with $P_s\to\infty, P_{r_1}\to\infty$ and $P_{r_2}\to\infty$. Also, assuming, $\frac{P_3^{t_{s3}}}{P_4^{t_{s3}}}=c$, $\frac{P_3^{t_{s3}}}{\sum_{j=1}^{2}P_j^{t_{s3}}+P_4^{t_{s3}}}=b$ and

$$\sigma^2\left(\frac{1}{(P_3^{t_{s_3}}-(\sum_{j=1}^{2}P_j^{t_{s_3}}+P_4^{t_{s_3}})y)P_s\lambda_1^{BS}}+\frac{1}{(P_3^{t_{s_3}}-P_4^{t_{s_3}}y)P_s\lambda_{r_2}^{BS}}+\frac{1}{(P_3^{t_{s_3}}-(\sum_{j=1}^{2}P_j^{t_{s_3}}+P_4^{t_{s_3}})y)P_s\lambda_{r_1}^{BS}}+\frac{1}{P_3^{t_{s_4}}P_{r_1}\lambda_3^{r_1}}\right)=a.$$

The ergodic rate of UE_3 in time sub-slot $t_{s_{3-4}}$ is:

$$E_3^{t_{s_{3-4}}}=\frac{t_{s_3}\times e}{log2}\left[Ei(c+1)+Ei(b+1)-2Ei(1)\right] \quad (116)$$

UE_4 Ergodic rate: the ergodic rate in the time slot $t_{s_{1-2}}$ is:

$$\begin{aligned}F_{SINR_{t_{s_{1-3}}}}^{4}(y)&=P\left(P_4^{t_{s_1}}\le y\right)=1-P\left(P_4^{t_{s_1}}>y\right)\\&=1-[P\left(SINR_{t_{s_1}}^{4-1}>y\right)P\left(SINR_{t_{s_1}}^{4-2}>y\right)P\left(SINR_{t_{s_1}}^{r_1}>y\right)P\left(SINR_{t_{s_2}}^{4}>y\right)\\&=\left[1-\left(1-F_{SINR_{t_{s_1}}^{4-1}}(y)\right)\left(1-F_{SINR_{t_{s_1}}^{4-2}}(y)\right)\left(1-F_{SINR_{t_{s_1}}^{r_1}}(y)\right)\left(1-F_{SINR_{t_{s_2}}^{4}}(y)\right)\right]\end{aligned} \quad (117)$$

$$F_{SINR_{t_{s_1}}^{4-2}}(y)=P\left(\frac{P_4^{t_{s_1}}P_s\left|h_2^{BS}\right|^2}{\sum_{j=1}^{3}P_j^{t_{s_1}}P_s\left|h_2^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s1}}-\sum_{j=1}^{3}P_j^{t_{s1}}y)P_s\lambda_2^{BS}} \quad (118)$$

$$F_{SINR_{t_{s_1}}^{r_1}}(y)=P\left(\frac{P_4^{t_{s_1}}P_s\left|h_{r_1}^{BS}\right|^2}{\sum_{j=1}^{3}P_j^{t_{s_1}}P_s\left|h_{r_1}^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s1}}-\sum_{j=1}^{3}P_j^{t_{s1}}y)P_s\lambda_{r_1}^{BS}} \quad (119)$$

$$F_{SINR_{t_{s_2}}^{4}}(y)=P\left(\frac{P_4^{t_{s_2}}P_{r_1}\left|h_4^{r_1}\right|^2}{\sigma^2}\le y\right)=1-e^{-y\sigma^2/P_4^{t_{s2}}P_{r_1}\lambda_4^{r_1}} \quad (120)$$

$$F_{SINR_{t_{s_{1-2}}}}^{4}(y)=\begin{cases}1-e^{-y\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_2^{BS}}+\frac{1}{P_s\lambda_{r_1}^{BS}}+\frac{1}{P_{r_1}\lambda_4^{r_1}}\right)}, 0<y<\frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3}P_j^{t_{s_1}}}\\ 1 \quad , else\end{cases} \quad (121)$$

For the closed-form with $P_s\to\infty, P_{r_1}\to\infty$ and $P_{r_2}\to\infty$. Also, assuming, $\frac{P_4^{t_{s_1}}}{\sum_{j=1}^{3}P_j^{t_{s_1}}}=b$ and $\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_2^{BS}}+\frac{1}{P_s\lambda_{r_1}^{BS}}+\frac{1}{P_{r_1}\lambda_4^{r_1}}\right)=a$. The ergodic rate of UE_4 in time sub-slot $t_{s_{1-2}}$ is:

$$E_4^{t_{s_{1-2}}}=-\frac{t_{s_1}}{log2}[Ei(a)-Ei(ab+a)]e^a \quad (122)$$

UE_4 Ergodic rate: the ergodic rate in the time slot t_{s_4} is:

$$F_{SINR_{t_{s_3}}^{4-1}}(y)=P\left(\frac{P_4^{t_{s_3}}P_s\left|h_1^{BS}\right|^2}{\sum_{j=1}^{2}P_j^{t_{s_3}}P_s\left|h_1^{BS}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s3}}-\sum_{j=1}^{2}P_j^{t_{s3}}y)P_s\lambda_1^{BS}} \quad (123)$$

$$F_{SINR_{t_{s3}}^{r_1\,4}}(y)=P\left(\frac{P_4^{t_{s3}}P_s\,|h_{r_2}^{BS}|^2}{\sum_{j=1}^{2}P_j^{t_{s3}}P_s\,|h_{r_2}^{BS}|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s3}}-\sum_{j=1}^{2}P_j^{t_{s3}})P_s\lambda_{r_2}^{BS}} \quad (124)$$

$$F_{SINR_{t_{s4}}^{4-2}}(y)=P\left(\frac{P_4^{t_{s4}}P_{r_2}\left|h_2^{r_2}\right|^2}{P_2^{t_{s4}}P_{r_2}\left|h_2^{r_2}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s4}}-\sum_{j=1}^{2}P_j^{t_{s4}})P_s\lambda_2^{r_2}} \quad (125)$$

$$F_{SINR_{t_{s4}}^{4}}(y)=P\left(\frac{P_4^{t_{s4}}P_{r_2}\left|h_4^{r_2}\right|^2}{P_2^{t_{s4}}P_{r_2}\left|h_4^{r_2}\right|^2+\sigma^2}\le y\right)=1-e^{-y\sigma^2/(P_4^{t_{s4}}-yP_2^{t_{s4}})P_{r_2}\lambda_4^{r_2}} \quad (126)$$

$$F_{SINR_{t_{s3-4}}}^{4}(y)=\begin{cases}1-e^{-y\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_{r_2}^{BS}}+\frac{1}{P_s\lambda_2^{r_2}}+\frac{1}{P_{r_2}\lambda_4^{r_2}}\right)}, & \frac{P_4^{t_{s3}}}{\sum_{j=1}^{2}P_j^{t_{s3}}}<y<\frac{P_4^{t_{s4}}}{P_2^{t_{s4}}}\\ 1 & ,else\end{cases} \quad (127)$$

For the closed-form with $P_s\to\infty, P_{r_1}\to\infty$ and $P_{r_2}\to\infty$. Also, assuming $\frac{P_4^{t_{s4}}}{P_2^{t_{s4}}}=c$, $\frac{P_4^{t_{s3}}}{\sum_{j=1}^{2}P_j^{t_{s3}}}=b$ and $\sigma^2\left(\frac{1}{P_s\lambda_1^{BS}}+\frac{1}{P_s\lambda_{r_2}^{BS}}+\frac{1}{P_s\lambda_2^{r_2}}+\frac{1}{P_{r_2}\lambda_4^{r_2}}\right)=a$.

The ergodic rate of UE_4 in time sub-slot $t_{s_{3-4}}$ is:

$$E_4^{t_{s3-4}}=-\frac{t_{s_3}}{log2}\left[\ln\left(\frac{c+1}{b+1}\right)+Ei\left(ac+a\right)-Ei(ab+a)e^{a}\right] \quad (128)$$

where $c-b>0$, $c+1>0$ and $b+1>0$.

The total ergodic rate of the system or the system ergodic sum rate is:

$$E=E_1^{t_{s1-2}}+E_1^{t_{s3-4}}+E_2^{t_{s1-2}}E_2^{t_{s3-4}}+E_3^{t_{s1-2}}+E_3^{t_{s3-4}}+E_4^{t_{s1-2}}+E_4^{t_{s3-4}} \quad (129)$$

Algorithm

Using the equation below the power coefficient matrix denoted by *pc_m* is generated:

$$\begin{pmatrix}P_1^{t_{s1}}+P_2^{t_{s1}}+P_3^{t_{s1}}+P_4^{t_{s1}}=1\\ P_3^{t_{s2}}+P_4^{t_{s2}}=1\\ P_1^{t_{s3}}+P_2^{t_{s3}}+P_3^{t_{s3}}+P_4^{t_{s3}}=1\\ P_2^{t_{s4}}+P_3^{t_{s4}}=1\end{pmatrix} \quad (130)$$

1. Initialize the number of users *n* and time sub-slots *k*. The power matrix $p\_m = zeros(n,k)$. The $pc\_m = zeros(n,k)$ and time matrix $t\_m = zeros(n,1)$.
2. Find the number of cell center users (n_c) and cell edge users (n_e)
3. Calculate $P_{n,t_{s_k}}$.
4 For $j = 1{:}n$
5. If $FI > FI_F$
6. If ($n_c > n_e$)
7. $FI = FI - a$, $0.5 \le a \le FI$
8 Else $n_c \ll n_e$
9. $p\_m(j,:) = pc\_m(j,:)$, $t\_m(j,:) = t_{s_j}$
10. End
11 . Else $FI < FI_F$
12. If $n_c < n_e$
13. $FI = FI + \left(FI - FI_F\right) = 2FI - FI_F$
14. $p\_m(j,:) = pc\_m(j,:)$, $t\_m(j,:) = t_{s_j}$
15. End
16. End
17. *Allocation Matrix* = *t_m p_m*

The algorithm checks the number of cell center users and cell edge users. When cell center users are more, the fairness index must be less than 0.5.

Simulation results

The simulation model is shown in Figures 1 and 2. The model has one BS, three relays (r_1, r_2, and r_3) and four users (UE_1, UE_2, UE_3 and UE_4). The relay works in half duplex mode. For t_{s_1} and t_{s_2} time sub-slots UE_1 and UE_2 are cell center users, while UE_3 and UE_4 are cell edge users using relays r_2 and r_1. For t_{s_3} and t_{s_4} time sub-slots UE_1 is a cell center user while UE_2, UE_3, and UE_4 are cell edge users. The UE_2 and UE_4 are connected to relay r_2. The UE_3 is connected to relay r_1. The value of P_s = 25 ~ 50 *dB*, P_{r_2} = 12.5 ~ 25 *dB* and P_{r_1} = $0.3P_s$. The channel variances for Figure 1 are $\lambda_1^{BS}=1^{-4}$, $\lambda_2^{BS}=0.8^{-4}$, $\lambda_{r_1}^{BS}=0.4^{-4}$, $\lambda_{r_2}^{BS}=0.5^{-4}$, $\lambda_3^{r_2}=0.3^{-4}$, and $\lambda_4^{r_1}=0.3^{-4}$. The channel variances for Figure 2 are $\lambda_1^{BS}=1^{-4}$, $\lambda_{r_1}^{BS}=0.4^{-4}$, $\lambda_{r_2}^{BS}=0.5^{-4}$, $\lambda_2^{r_2}=0.3^{-4}$, $\lambda_3^{r_1}=0.3^{-4}$, and $\lambda_4^{r_2}=0.4^{-4}$. The target rate $R=0.3$ *bps/Hz* and the fairness index factor is FI_F. The power allocation algorithm is from Fang et al. (2017). The users theoretical and simulated rates are compared in Figure 3 for FI_F = 0.5. The theoretical and simulated values of rates are closer to each other. An increase in transmission power of each user increases the corresponding rates. The rates of user UE_1 increases in all the time sub-slots as it is always a cell center user. The situation for UE_2 changes in time sub-slots t_{s_3} and t_{s_4} when it becomes a cell edge user which leads to a lower growth rate in these sub-slots. For UE_3 and UE_4, the growth of rate is slow in all time sub-slots as they are always cell edge users.

The user's outage probability and transmission power are compared in Figure 4. The use of different transmission power requires different power allocation adopted from (Fang et al., 2017). The plot

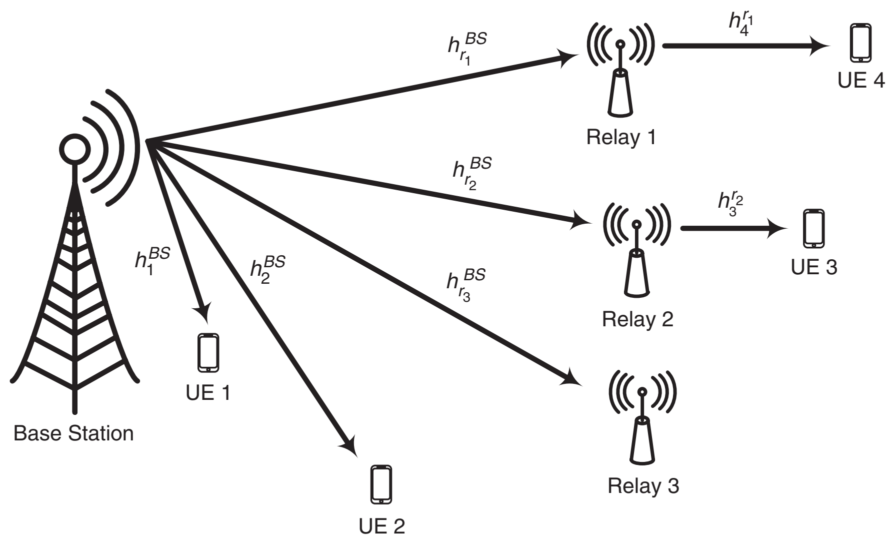

Figure 1: System Model 1.

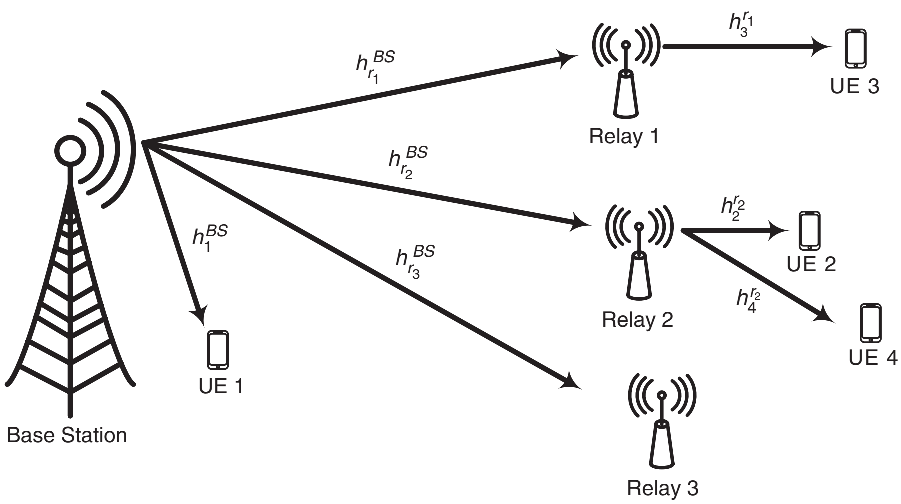

Figure 2: System Model 2.

of outage probability with transmission power is non-linear for fairness $FI_F = 0.5$.

The outage probability of the system is compared with the equal time transmission approach in Figure 5. When transmission power is lower, the outage probability of the proposed system is low, while the performance of the two systems is closer when the transmission power is high. The user rates of the system are compared with the variation in the fairness index in Figure 6 with a maximum transmission power of all devices. The rates of cell center users decrease with the increase in the fairness index while the rates of cell edge users increase. For UE_1 rate decreases with an increase in FI. For UE_2 in the first two time sub-slots the rate decreases and as it moves from position of relay user to single hop communication user. The system rate also decreases at a lesser rate in the last two sub-slots.

Conclusion

In this paper, relays are used for cell edge users which improves spectral efficiency for NOMA network with relays. Initially, the scenario of two-hop

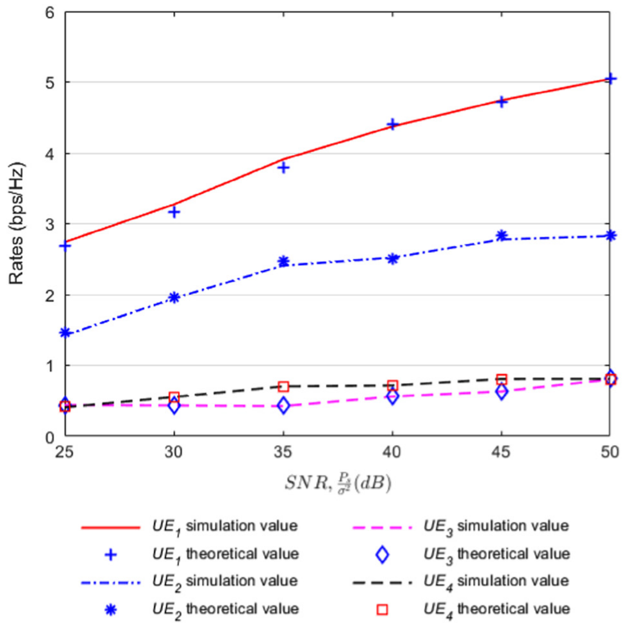

Figure 3: Users theoretical and simulated results.

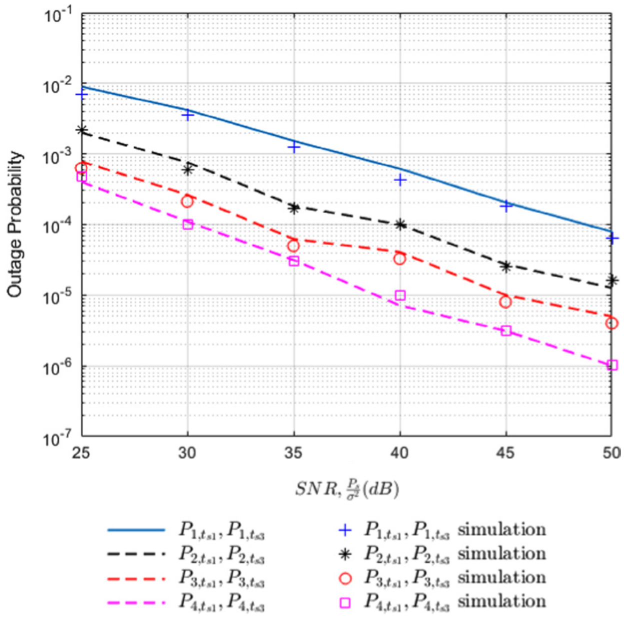

Figure 4: User's outage probability and transmission power.

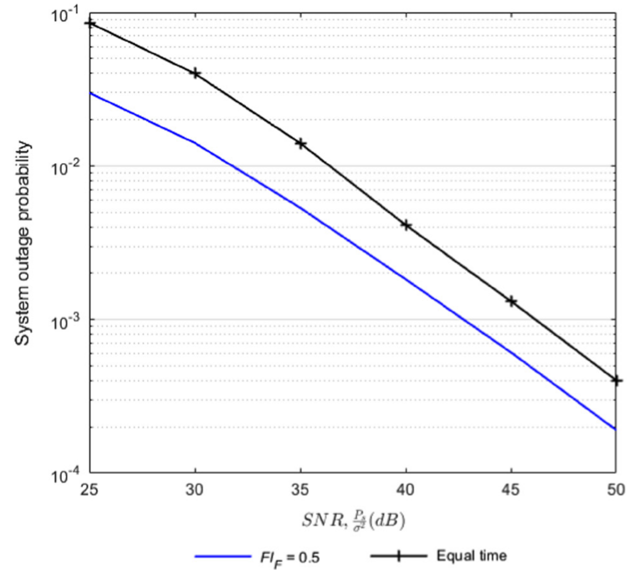

Figure 5: Outage probability of the system.

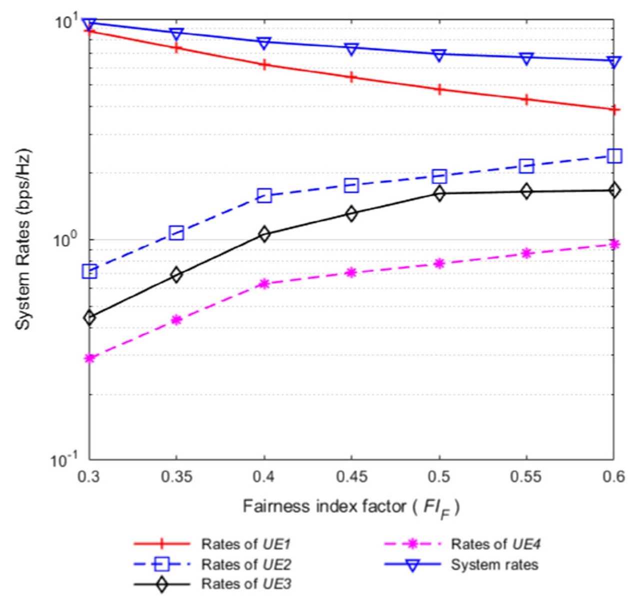

Figure 6: User rate and fairness index.

communication is explained in the time sub-slots. The relays are used for two-hop communication which reduces the outage probability. The expressions for outage probability and ergodic rates considering the mobility of the users during one-time slot are derived. The simulations are done with fairness and without fairness (equal time), considering its effect on both cell center and cell edge users. The proposed algorithm improves the system's rate significantly during the consideration of fairness among users. In future work, can be done considering multi hops and reducing complexity associated with them.

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Literature Cited

Al-Abbasi, Z. Q. and So, D. K. C. 2016. User-pairing based non-orthogonal multiple access (NOMA) system. *2016 IEEE 83rd Vehicular Technology Conference* (*VTC Spring*) 2016: 1–5.

Aldababsa, M., Toka, M., Gökçeli, S., Kurt, G. K. and Kucur, O. 2018. A tutorial on nonorthogonal multiple access for 5G and beyond. *Wireless Communications and Mobile Computing* 2018: 1–24.

Balyan, V. 2020. Outage probability of cognitive radio network utilizing non orthogonal multiple access. *2020 7th International Conference Signal Processing and Integrated Networks* (*SPIN*), Noida, pp. 751–755.

Bonnefoi, R., Fares, H., Belis, P. and Louet, Y. 2019. Optimal power allocation for minimizing the energy consumption of a NOMA base station with Cell DTx. *2019 URSI Asia-Pacific Radio Science Conference* (*AP-RASC*), New Delhi pp. 1–4.

Cheng, M., Wu, Y. and Chen, Y. 2015. Capacity analysis for non-orthogonal overloading transmissions under constellation constraints. *2015 International Conference on Wireless Communications & Signal Processing* (*WCSP*), 1–5.

Choi, J. 2017. Effective capacity of NOMA and a suboptimal power control policy with delay QoS. *IEEE Transactions on Communications* 65(4): 1849–1858.

Di, B., Song, L. and Li, Y. 2016. Sub-channel assignment, power allocation, and user scheduling for non-orthogonal multiple access networks. *IEEE Transactions on Wireless Communications* 15(11): 7686–7698.

Ding, Z., Adachi, F. and Poor, H. V. 2016. The application of MIMO to non-orthogonal multiple access. *IEEE Transactions on Wireless Communications* 15(1): 537–552.

Fang, F., Zhang, H., Cheng, J. and Leung, V. C. M. 2017. Energy-efficient resource scheduling for NOMA systems with imperfect channel state information. 2017 *IEEE International Conference on Communications* (*ICC*) 2017: 1–5.

Fang, F., Zhang, H., Cheng, J., Roy, S. and Leung, V. C. M. 2017. Joint user scheduling and power allocation optimization for energy-efficient NOMA systems with imperfect CSI. *IEEE Journal on Selected Areas in Communications* 35(12): 2874–2885.

He, J., Tang, Z. and Che, Z. 2016. Fast and efficient user pairing and power allocation algorithm for non-orthogonal multiple access in cellular networks. *Electronics Letters* 52(25): 2065–2067.

Manglayev, T., Kizilirmak, R. C. and Kho, Y. H. 2016. Optimum power allocation for non-orthogonal multiple access (NOMA). *2016 IEEE 10th International Conference on Application of Information and Communication Technologies* (*AICT*) 2016: 1–4.

Nasser, A., Muta, O., Elsabrouty, M. and Gacanin, H. 2019. Interference mitigation and power allocation scheme for downlink MIMO–NOMA HetNet. *IEEE Transactions on Vehicular Technology* 68(7): 6805–6816.

Oviedo, J. A. and Sadjadpour, H. R. 2018. On the power allocation limits for downlink multi-user NOMA with QoS. *2018 IEEE International Conference on Communications* (*ICC*) 2018: 1–5.

Shi, J., Yu, W., Ni, Q., Liang, W., Li, Z. and Xiao, P. 2019. Energy efficient resource allocation in hybrid non-orthogonal multiple access systems. *IEEE Transactions on Communications* 67(5): 3496–3511.

Shrestha, A. P., Han, T., Bai, Z., Kim, J. M. and Kwak, K. S. 2016. Performance of transmit antenna selection in non-orthogonal multiple access for 5G systems. *2016 Eighth International Conference on Ubiquitous and Future Networks* (*ICUFN*) 2016(August): 1031–1034.

Sun, Q., Han, S., I, C. -L. and Pan, Z. 2015. On the ergodic capacity of MIMO NOMA systems. *IEEE Wireless Communications Letters* 4(4): 405–408.

Timotheou, S. and Krikidis, I. 2015. Fairness for non-orthogonal multiple access in 5G systems. *IEEE Signal Processing Letters* 22(10): 1647–1651.

Xu, P. and Cumanan, K. 2017. Optimal power allocation scheme for non-orthogonal multiple access with α-fairness. *IEEE Journal on Selected Areas in Communications* 35(10): 2357–2369.

Yang, Z., Ding, Z., Fan, P. and Al-Dhahir, N. 2016. A general power allocation scheme to guarantee quality of service in downlink and uplink NOMA systems. *IEEE Transactions on Wireless Communications* 15(11): 7244–7257.

Yuan, Z., Li, X. and Lv, G. 2019. Energy efficient power allocation for multi-carrier non-orthogonal multiple access (NOMA) systems with proportional rate constraints. *2019 IEEE 3rd Information Technology, Networking, Electronic and Automation Control Conference* (*ITNEC*) No. ITNEC, 1261–1265.

Zeng, M., Hao, W., Dobre, O. A. and Poor, H. V. 2019. Energy-efficient power allocation in uplink mmWave massive MIMO with NOMA. *IEEE Transactions on Vehicular Technology* 68(3): 3000–3004.

Zhang, X., Zhu, X. and Zhu, H. 2019. Joint user clustering and multi-dimensional resource allocation in downlink MIMO–NOMA networks. *IEEE Access* 7: 81783–81793.

Zhao, S., Mei, C. and Zhu, Q. 2019. Joint time and power allocation algorithm in NOMA relaying network. *Mobile Informations Systems* 2019: 1–14.