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A comparative study of a class of direct estimators for domain mean with a direct ratio estimator for domain mean using auxiliary character

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ABSTRACT

Estimation techniques for a domain parameter play a very significant role in the theory of sample surveys. In the recent years many advanced methodologies have been developed for domain estimation. In particular, direct and synthetic estimators are applied for the estimation of domain mean in the government and private sectors under certain assumptions as to the size of the samples relating to particular domains. The findings demonstrate that the direct estimator fails to perform more efficiently as compared to the synthetic estimator when reliable units are not directly accessible in the studied domains. Moreover, due to the fact that small units belong to the sample of the studied domain, the direct estimator produces an unacceptably large standard error. In contrast, if a sufficient number of units are available in the studied domain, the direct estimator produces effective results. This paper presents the theoretical aspects of the proposed class of direct estimators for domain mean with the use of a single auxiliary character, compared with an existing direct ratio estimator for domain mean (given in section 3.2). In addition, an empirical study has been provided to support the validity of the proposed estimators. The findings prove that the proposed estimators outperform the direct ratio estimator for domain mean using a single auxiliary character in the case of two studied populations and their analysed domains considered from Sarndal et al. (1992).

Key words: domain, auxiliary character, direct ratio estimator, class of estimators, mean square error (MSE).

1. Introduction

If we are interested in the estimation of subpopulations also called domains like a block, a county and a village, etc., instead of whole population. It has been seen in recent years that the accelerated demand for policy implementation and decision-

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makers, different types of estimation methods have been developed, which may solve these types of problems. There are two types of methods direct method and indirect method of estimations are used in the estimation of domain parameter. The direct method is generally used when the number of sufficient units is accessible in the study domain. In the direct method, we take a sample from the study domain and it is applied in the estimator which may improve the efficiency of the estimator. The estimator based on the sole of direct method has been illustrated in the book by Rao (2003). Whenever, accessible numbers of units do not sufficiently in the study domain, we prefer an indirect method based estimator. In this situation, a sample is selected from the whole population instead of subpopulation. Eminent works have been already done using the indirect method of estimation, e.g. synthetic estimators using auxiliary character have been illustrated by Gonzalez (1973), Tikkiwal and Ghiya (2000), Rai and Pandey (2013) and Khare and Ashutosh (2017), among many others in sample surveys. We mainly collect the fact from the surrounding value of auxiliary and study information and employ them in the estimator to improve the efficiency of the estimator. An idea of class of estimator to estimate the population mean has been given by Srivastava (1971).

In this paper, he proposed an estimator $\bar{y}_h = \bar{y}h(u)$, where $u = \frac{\bar{x}}{\bar{X}}$, under certain regularity conditions, the asymptotic MSE is same for all its members. In his unpublished work, he did the extension of his own above paper of a wider class of estimator, which is $\bar{y}_g = g\left(\bar{y}, \frac{\bar{x}}{\bar{X}}\right)$ where the function $g(.,.)$ satisfies the regularity conditions. Furthermore, another work related to the class of estimator for population mean was discussed by Srivastava and Jhajj (1981). They also developed a class of estimator for finite population mean using single auxiliary character x according to some parametric function h(.), which satisfied certain regularity conditions along with the limitation of h_1=1, and also it was shown that the lower bound of the asymptotic MSE of the estimator is equal as the asymptotic MSE of the linear regression estimator, which is itself not a member of the class of estimator developed by Srivastava and Jhajj (1981). Another work was done by Srivastava (1983) which may get an improve version of the above paper, in which he incorporated another parameters called variance, and he suggested $\bar{y}_g = g(\bar{y}, u, v)$ where $u = \frac{\bar{x}}{\bar{X}}$ and $v = \frac{s_x^2}{S_x^2}$.

where, $\bar{y}$ = Sample mean of study character,

$\bar{x}$ = Sample mean of auxiliary character,

$\bar{X}$ = Population mean of auxiliary character,

s_x^2 = Sample mean square of auxiliary character x and

S_x^2 =Population mean square of auxiliary character x.

Other works have also been done which are related to the class of estimator using two phase sampling scheme for estimation of the population mean discussed by Srivastava and Khare (1993), Khare and Pandey (2000), and Khare and Sinha (2009). In the coming year work has been done by Khare et al. (2018). They have proposed a class of synthetic estimators for domain parameter like mean, and a function which combined value of u and v, the estimator is given by

$$T_{SC,a} = f(u, v)$$

where $u = \bar{y}$ and $v = \frac{\bar{x}}{\bar{X}_a}$,

$\bar{y}$ = Sample mean of population of study character.

$\bar{x}$ = Sample mean of population of auxiliary character.

$\bar{X}_a$ = Population mean of ath Domain of auxiliary character.

In the present paper, we obtained MSE of the members of the proposed estimators for domain mean $\bar{Y}_a$ is equal under the certain regularity conditions but their values of constants are different. And we proposed a class of direct estimator for domain mean using auxiliary character, which is given by

$$T_{D,C,a} = h(u, v)$$

where $u = \bar{y}_a$ and $v = \frac{\bar{x}_a}{\bar{X}_a}$, which is given in the further section.

The particular cases of the proposed class of direct estimator are also discussed for domain mean. A comparative study of the proposed estimator for domain mean $(T_{D,C,a})$ with direct ratio estimator for domain mean $(T_{D,RS,a})$ has been given by using the real data of Swedish municipalities (Sarndal et al. (1992)).

2. Formulation of the problem and notations for domains

Suppose that non-overlapping domains U_a of size N_a such that (*a*=1, 2, 3,.., A). Now, our interest is in the estimation of the parameter of the domain mean $\bar{Y}_a$ of ath domain with size N_a. Later, we selected a sample $'s'$ through simple random sampling without replacement (SRSWOR) in which come from a^{th} domain have size n_a from domain population size N_a. We represent the study character and auxiliary character by y and x respectively.

We denote the population mean and sample mean for domain of y and x as follows:

$\overline{Y}_a$: a^{th} domain meanof y based on size N_a observations.

$\overline{y}_a$: Sample mean of y of a^{th} domain based on n_a observations.

$\overline{X}_a$: a^{th} domain mean of x based on size N_a observations.

$\overline{x}_a$: Sample mean of x of a^{th} domain based on n_a observations.

Let us denote y_{a_i} as the i^{th} observation of a^{th} domain of the study character y for domain U_a ($a = 1, 2, ..., A,\ i = 1, 2, ..., N_a$) and x_{a_i} is the i^{th} observation of a^{th} domain of the auxiliary character x for domain U_a ($a = 1, 2, ...A,\ i = 1, 2, ..., N_a$).

We further use the following notations:

$$S_{Y_a}^2 = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} \left(y_{a_i} - \overline{Y}_a\right)^2, \quad S_{X_a}^2 = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} \left(x_{a_i} - \overline{X}_a\right)^2,$$

$$S_{X_aY_a} = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} \left(x_{a_i} - \overline{X}_a\right)\left(y_{a_i} - \overline{Y}_a\right), \quad C_{Y_a} = \frac{S_{Y_a}}{\overline{Y}_a},$$

$$C_{X_a} = \frac{S_{X_a}}{\overline{X}_a} \text{ and } C_{X_aY_a} = \frac{S_{X_aY_a}}{\overline{X}_a \overline{Y}_a}. \tag{2.1}$$

3. Direct estimator for domain mean using single auxiliary character

There are several direct estimators which are used for estimation of population parameters of different segments. Auxiliary characteristics are used to improve the existing estimator for the domains. Here, in our case, we are considering the direct ratio estimator for estimating domain mean. Thus, let us consider the case of the direct ratio estimator under the above design and obtain the expressions of Bias and MSE in the next subsection.

3.1. Direct ratio estimator for domain mean

$$T_{D,RS,a} = \frac{\overline{y}_a}{\overline{x}_a} \overline{X}_a \quad \text{Tikkiwal and Ghiya (2000)} \tag{3.1.1}$$

$$Bias(T_{D,RS,a}) = \frac{(N_a - n_a)}{N_a n_a} \overline{Y}_a (C_{X_a}^2 - C_{X_aY_a}) \tag{3.1.2}$$

$$MSE(T_{D,RS,a}) = \frac{(N_a - n_a)}{N_a n_a} \bar{Y}_a^2 (C_{Y_a}^2 + C_{X_a}^2 - 2C_{X_a Y_a}) \tag{3.1.3}$$

3.2. proposed class of direct estimators for domain mean using single auxiliary character $T_{D,C,a}$

We proposed a class of direct estimators for domain using auxiliary character, which is given as:

$$T_{D,C,a} = h(u,v) \tag{3.2.1}$$

where, $u = \bar{y}_a$ and $v = \frac{\bar{x}_a}{\bar{X}_a}$ and the function $h(u,v)$ satisfied the following regularity conditions:

1. The function $h(u,v)$ exists for all the values of (u,v) and it contains the points $(\bar{Y}_a, 1)$ in a bounded subset D of two dimensional real spaces.
2. The first and second order partial derivatives of $h(u,v)$ exist and are bounded also.

Members of the estimator for C=1, 2 and 3 are given as follows:

$$T_{D,1,a} = uv^{\alpha} \tag{3.2.2}$$

$$T_{D,2,a} = \alpha_1 u + (1-\alpha_1) u v^{\alpha_2} \tag{3.2.3}$$

$$T_{D,3,a} = ue^{\alpha 3 \frac{(v-1)}{(v+1)}} \tag{3.2.4}$$

Now, expanding the proposed class of estimators $T_{D,C,a}$ using Taylor series expansion about the point $(\bar{Y}_a, 1)$ up to the second order, we have

$$T_{D,C,a} = h(u,v)_{(\bar{Y}_a,1)} + (u-\bar{Y}_a)h_1 + (v-1)h_2 + + \frac{1}{2}\left[(u-\bar{Y}_a)^2 + (v-1)^2 h_{22} + (v-1)h_{12}\right] \tag{3.2.5}$$

where

$$h_1 = \left(\frac{\partial h(u,v)}{\partial u}\right)_{(\bar{Y}_a,1)}, h_2 = \left(\frac{\partial h(u,v)}{\partial v}\right)_{(\bar{Y}_a,1)}, h_{11} = \left(\frac{\partial^2 h(u,v)}{\partial u^2}\right)_{(\bar{Y}_a,1)},$$

$$h_{22} = \left(\frac{\partial^2 h(u,v)}{\partial v^2}\right)_{(\bar{Y}_a,1)}$$

and $h_{12} = \left(\frac{\partial^2 h(u,v)}{\partial u \partial v}\right)_{(\bar{Y}_a,1)}$.

(3.2.6)

Now, we put h_1=1 and $h_{11} = 0$ in the equation (3.2.5), and we have

$$T_{D,C,a} = h(u,v)_{(\overline{Y}_a,1)} + (v-1)h_2 + \frac{1}{2}[(v-1)^2 h_{22} + 2(v-1)(u-\overline{Y}_a)h_{12}]. \tag{3.2.7}$$

For large sample approximations, we assume that

$$\overline{y}_a = \overline{Y}_a(1+\varepsilon_0), \overline{x}_a = \overline{X}_a(1+\varepsilon_1), \text{ such that } E(\varepsilon_0)=0, E(\varepsilon_1)=0,$$

$$E(\varepsilon_0^2) = \frac{(N_a - n_a)}{N_a n_a} C_{Y_a}^2 \; E(\varepsilon_1^2) = \frac{(N_a - n_a)}{N_a n_a} C_{X_a}^2 \text{ and}$$

$$E(\varepsilon_0 \varepsilon_1) = \frac{(N_a - n_a)}{N_a n_a} C_{X_a Y_a}. \tag{3.2.8}$$

The Bias and MSE of the proposed class of the estimators for domain mean using auxiliary character is obtained as:

$$Bias(T_{D,C,a}) = \frac{(N_a - n_a) S_{X_a}^2 h_2}{2 N_a n_a \overline{X}_a^2} \left(\frac{h_2}{\overline{Y}_a} - 1 \right) + \frac{(N_a - n_a) S_{X_a Y_a}}{N_a n_a \overline{X}_a \overline{Y}_a} h_2 \tag{3.2.9}$$

$$MSE(T_{D,C,a}) = E\left[(\overline{y}_a - \overline{Y}_a)^2 + \left(\frac{\overline{x}_a}{\overline{X}_a} - 1 \right) h_2 + \frac{1}{2} \left\{ \left(\frac{\overline{x}_a}{\overline{X}_a} - 1 \right)^2 h_2^2 + 2 \left(\frac{\overline{x}_a}{\overline{X}_a} - 1 \right) (\overline{y}_a - \overline{Y}_a) \frac{h_2}{\overline{Y}_a} \right\} \right] \tag{3.2.10}$$

Now, for optimum value of h_2, we partially differentiate equation (3.2.10) w.r.to h_2 and equating to zero, we have

$$h_{2,opt} = -\frac{\overline{X}_a \rho_a \sqrt{S_{Y_a}^2}}{\sqrt{S_{X_a}^2}} \tag{3.2.11}$$

After substituting the value of $h_{2,opt}$ in the equation (3.2.10) the optimum MSE of $T_{D,C,a}$ is given by

$$MSE(T_{D,C,a,opt}) = (1 - \rho_a^2) \frac{(N_a - n_a)}{N_a n_a} S_{Y_a}^2 \tag{3.2.12}$$

Theorem 1. The values of the constants (given in equations 3.2.2, 3.2.3 and 3.2.4) of the member of the proposed estimators, which are included in $T_{D,1,a}$, $T_{D,2,a}$ and $T_{D,3,a}$ after minimizing their individual MSE expressions, are given as follows:

$$\alpha_{opt} = -\frac{\overline{X}_a \rho_a \sqrt{S^2_{Y_a}}}{\overline{Y}_a \sqrt{S^2_{X_a}}} \tag{3.2.13}$$

$$\alpha_{2,opt} = -\frac{\overline{X}_a \rho_a \sqrt{S^2_{Y_a}}}{(1-\alpha_1)\overline{Y}_a \sqrt{S^2_{X_a}}} \quad \text{where,} 0<\alpha_1<1 \tag{3.2.14}$$

$$\alpha_{3,opt} = -\frac{2\overline{X}_a \rho_a \sqrt{S^2_{Y_a}}}{\overline{Y}_a \sqrt{S^2_{X_a}}} \tag{3.2.15}$$

The minimum values of the MSE of the estimators $T_{D,1,a}$, $T_{D,2,a}$ and $T_{D,3,a}$ for optimum values of the constants α_{opt}, $\alpha_{2,opt}$ and $\alpha_{3,opt}$ are the same and given in the equation (3.2.12), the optimum value of the constants α_{opt}, $\alpha_{2,opt}$ and $\alpha_{3,opt}$ are given in the form of the parameters in the equations (3.2.13), (3.2.14) and (3.2.15), it may be possible use of the optimal values using the past data regarding parameters given by Reddy (1978), and it has been seen that in the terms of order n^{-1}, the minimum value of the MSE of the estimator does not change when we estimate the optimal value of the constants using the sample values of idea given by Srivastava and Jhajj (1981).

3.3. Comparison between proposed class of estimators and direct ratio estimator for domain mean using auxiliary character

Let us consider $MSE(T_{D,RS,a}) - MSE(T_{D,C,a,opt}) \geq 0$

$$= \frac{(N_a - n_a)}{N_a n_a}\overline{Y}_a^2\left(C^2_{Y_a} + C^2_{X_a} - 2C_{X_aY_a}\right) - \left(1-\rho_a^2\right)\frac{(N_a - n_a)}{N_a n_a}S^2_{Y_a}$$

$$= \frac{(N_a - n_a)}{N_a n_a}\overline{Y}_a^2\left(C^2_{X_a} - 2C_{X_aY_a} + \rho_a^2 C^2_{Y_a}\right)$$

$$= \frac{(N_a - n_a)}{N_a n_a}\overline{Y}_a^2\left(\rho_a C_{Y_a} - C_{X_a}\right)^2 \tag{3.3.1}$$

Since $\left(\rho_a C_{Y_a} - C_{X_a}\right)^2$ must be positive. Hence,

$$MSE(T_{D,RS,a}) - MSE(T_{D,C,a,opt}) \geq 0$$

$$\text{i.e. } MSE(T_{D,RS,a}) \geq MSE(T_{D,C,a,opt}) \tag{3.3.2}$$

4. Empirical study

For the purpose of empirical study, we considered the data from Sarndal et al. (1992) in the appendix B. The population of Swedish municipalities is classified into eight non-overlapping domains, but we consider only four domains i.e. 2, 3, 4 and 5 have sizes (48, 32, 38 and 56). The empirical study of the two populations (1 and 2), the information about population 1 and population 2 is given as follows:

Population 1

y= Revenues from the 1985 municipal taxation (in millions of kronor)

x= Real estate values according to 1984 assessment (in millions of kronor).

Table 4.1. The parameter values of the domains (1, 2, 3 and 4)

Domain Values	Domain			
	1	2	3	4
N_a	48	32	38	56
$\overline{Y}_a$	233.69	176.13	265.74	273.30
$\overline{X}_a$	2970.958	2498.75	2915.526	3046.946
$S^2_{X_a}$	11118969	4474735	27860176	27861139
$S^2_{Y_a}$	93788.43	32183.08	311726.60	788518.90
$S_{X_aY_a}$	990772.90	344998.30	1621192.00	4518431.00
ρ_a	0.970	0.942	0.938	0.964

Population 2

y=Real estate values according to 1984 assessment (in millions of kronor).

x= Number of municipal employees in 1984.

Table 4.2. The parameter values of the different domains (1, 2, 3 and 4)

Domain Values	Domain			
	1	2	3	4
N_a	48	32	38	56
$\overline{X}_a$	1658.708	1316.938	1937.5	1950. 393
$\overline{Y}_a$	2970.96	2498.75	2915.53	3046.95
$S^2_{X_a}$	4601899	1989177	15986523	38786393
$S^2_{Y_a}$	11118969	4164522	27860176	27861139
$S_{X_aY_a}$	6920432	2681882	11697923	31770622
ρ_a	0.967414	0.9317951	0.9454677	0.9664827

Table 4.3. MSE of the direct ratio estimator for domain mean using auxiliary character ($T_{D,RS,a}$) and MSE of the proposed estimators for domain mean using auxiliary character ($T_{D,C,a,opt}$) for the optimum values of $h_{2,opt}$ for all domains 1, 2, 3 and 4, also the value of different constants which exist in the proposed estimators (**for population 1**):

Estimator		**Domains**			
		1	**2**	**3**	**4**
$T_{D,RS,a}$		1203.66	1786.603	21417.37	36810.38
$T_{D,C,a,opt}$		986.148	1088.301	8333.17	10151.570
$h_{2,opt}$		-264.732	-207.002	-493.607	-494.144
α		-1.133	-1.175	-1.857	-1.808
α_1	0.25	(-1.510)	(-1.567)	(-1.896)	(-2.411)
	0.50	(-2.266)	(-2.351)	(-3.715)	(-3.616)
	0.75	(-4.531)	(-4.701)	(-7.430)	(-7.232)
α_3		-2.266	-2.351	-3.715	-3.616

() shows α_2 constant included in the estimator ($T_{D,2,a}$)

Table 4.4. MSE of the direct ratio estimator for domain mean using auxiliary character ($T_{D,RS,a}$) and the proposed estimators for domain mean using auxiliary character ($T_{D,C,a,opt}$) at optimum values of function $h_{2,opt}$ for all domains 1, 2, 3 and 4, also the value of constants which are included in the member of the estimators for population 2:

Estimator		**Domains**			
		1	**2**	**3**	**4**
$T_{D,RS,a}$		61034.97	96377.97	1044100	1735007
$T_{D,C,a,opt}$		52858.67	79172.95	379373.3	465640
$h_{2,opt}$		-1189.497	-1112.12	-4602.01	-4099.073
α		-0.717	-0.845	-2.375	-2.102
α_1	0.25	(-0.956)	(-1.126)	(-3.167)	(-2.802)
	0.50	(-1.434)	(-1.689)	(-4.750)	(-4.203)
	0.75	(-2.868)	(-3.378)	(-9.501)	(-8.407)
α_3		-1.434	-1.690	-4.751	-4.203

() shows α_2 constant included in estimator ($T_{D,2,a}$)

From the table (4.3) it is seen that the amount of MSE of the class of direct estimators for domain mean ($T_{D,C,a,opt}$) is less than the amount of MSE of the direct ratio estimator for domain mean ($T_{D,RS,a}$) for domain 1 and the value of $h_{2,opt}$=-264.732 but the value of the member of the constants α =-1.133, α_1 =0.25, α_2 =-1.510 and α_3 =-2.266 is different, and for domain 2, 3, and 4, the value of $h_{2,opt}$ is fixed while the value of constant is different for population 1.

From the table (4.4) it is seen that the amount of MSE of the class of direct estimator for domain mean ($T_{D,C,a,opt}$) is less than the amount of MSE of the direct ratio estimator for domain mean ($T_{D,RS,a}$) for domain 1 and the value of $h_{2,opt}$ =-1189.497 but the value of the member of the constants α =-0.717, α_1 =0.25, α_2 =-0.956 and α_3 =-1.434 is different. This pattern is also seen for others domains 2, 3 and 4 for population 2.

Table 4.5. Percentages Relative Efficiency (PRE) of the proposed estimator for domain mean ($T_{D,C,a,opt}$) to direct ratio estimator for domain mean ($T_{D,RS,a}$) for different domains 1, 2, 3 and 4 (**for population 1 and population 2**):

Population	**Estimators**	**Domains**			
		1	**2**	**3**	**4**
1	$T_{D,RS,a}$	100.000	100.000	100.000	100.000
	$T_{D,C,a,opt}$	122.057	164.164	257.014	362.608
2	$T_{D,RS,a}$	100.000	100.000	100.000	100.000
	$T_{D,C,a,opt}$	115.468	121.731	275.217	372.607

From the table (4.5), it is observed that the value of PRE of the proposed estimator for domain mean ($T_{D,C,a,opt}$) is higher than the PRE of the direct ratio estimator ($T_{D,RS,a}$) for all domains 1, 2, 3 and 4 for population 1 and population 2.

5. Conclusion and recommendations

It is emphasized that the MSE of the proposed class of estimators for domain mean is less than the corresponding MSE of the direct ratio estimator for the domain mean in both populations considered for empirical analysis for nearly all domains. Also, the results for MSE supported the superiority of the proposed estimators theoretically as compared to the direct ratio estimator. There are some deviations in the result of MSE of the proposed estimator for the first and second domains as compared to the results for the third and fourth domains. It may be due to the variation present in the observations. PRE is also calculated for the proposed estimator and derived the results for family of estimators under certain regularity conditions given in the literature. Also, it is shown that the values of three constants available in the proposed member of family of estimators are different while the function $h_{2\,opt}$ is fixed under certain regularity conditions in the domains for both first and second populations considered for analysis.

Thus, it is recommended that the class of direct estimators proposed in this article for the estimation of domain mean using proper auxiliary information have substantial utility in the domain estimation methodology as compared to the existing direct ratio estimator under the condition that a sufficient member of units fall in the domain concerned.

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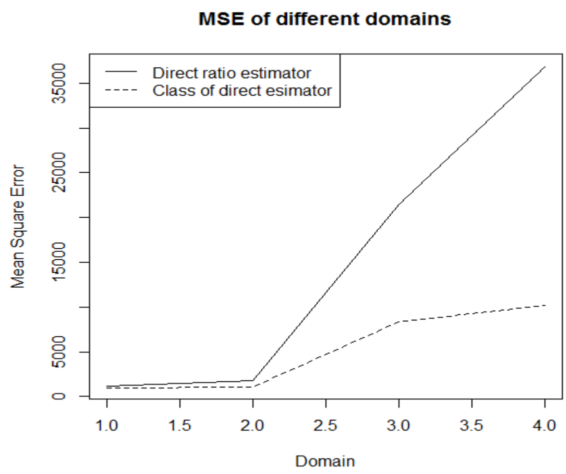
APPENDIX

Population 1

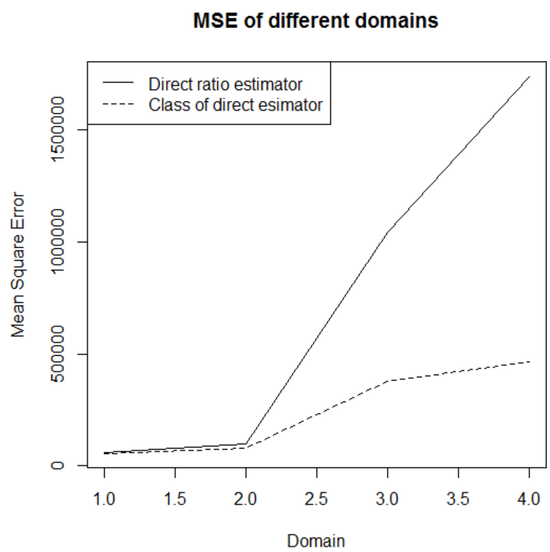
Figure1. Mean Square Error for different domains (1,2,3 and 4)



y axis: Mean Square Error, x axis: Domain

Population 2

Figure 2. Mean Square Error for different domains (1,2,3 and 4)



y axis: Mean Square Error, x axis: Domain