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THE OPTIMIZATION APPROACH OF POSTAL TRANSPORTATION NETWORK BASED ON UNCAPACITATED FIXED CHARGE LOCATION MODEL IN CONDITIONS OF SLOVAK REPUBLIC

Summary. The article deals with the possibilities of optimizing the postal transportation network with respect to planned road infrastructure. The research adopted in this article uses allocation models within graph theory to obtain results for addressed optimization problem. The article presents and compares two types of these models – p-median and uncapacitated fixed charge facility location model. The latter is subsequently applied on the postal network to determine the optimal location of postal facilities while minimizing costs. Moreover, the article describes the possibilities of identifying and calculating input variables of the used model, creating the underlying network, as well as possible further improvements of obtained solution. The results can serve as a basis for modification of the used model for the simulation of networks in the postal sector.

OPTIMALIZACJA PODEJŚCIA DO POCZTOWEJ SIECI TRANSPORTOWEJ NA PODSTAWIE MODELU W WARUNKACH SŁOWACJI

Streszczenie. Artykuł dotyczy możliwości optymalizacji pocztowej sieci transportowej w odniesieniu do planowanej infrastruktury drogowej. Do uzyskania wyników badań dla problemu optymalizacji wykorzystane zostały modele alokacji w ramach teorii grafów. Artykuł przedstawia i porównuje dwa rodzaje modeli. Model wykorzystuje się w celu określenia optymalnego położenia urządzeń wyposażenia pocztowego, przy jednoczesnej minimalizacji kosztów. Ponadto, artykuł opisuje możliwości określania i obliczania zmiennych wejściowych używanego modelu, tworzenia sieci podstawowej, jak również możliwych dalszych usprawnień otrzymanego rozwiązania. Wyniki badań mogą służyć jako podstawa do modyfikacji używanych modeli symulacji w sieciach sektora pocztowego.

1. INTRODUCTION

Decision of facilities location is an integral part of public and private sector. For the public sector this issue may represent the location of ambulance or fire stations. Incorrect position in network in such case increases the probability of health and property damage, even loss of life. In private sector it may be a decision concerning the placement of warehouses, offices or distribution centers within related network [1, 2]. Here the incorrect location increases costs and reduces competitiveness. We may conclude that the success or failure of facilities in both sectors also depends on the location of facility itself. This issue is addressed in so-called *allocation problems* [3].

There are several methods for modeling and solving allocation problems. The most important are methods of integer linear programming and graph theory. Allocation problems differ in type of objective function and model of the environment in which they are addressed. Model of environment, in our case, is the transportation network abstracted by the complete weighted graph $G = (V, E, c, w)$. V is the set of vertices representing possible facility locations. E is the set of edges representing connections between nodes (vertices). Label $c(e)$ of edge $e \in E$ is its length. Weight $w(v)$ of node $v \in V$ represents the importance of node in addressed system (number of demands, etc.) [4].

Some nodes can serve as centers (hubs). These centers can generally have two functions – *rescue or supply*. When speaking about rescue function, the center is called the emergency center. This apply for ambulance locations, etc. The significant criterion in this case is the reachability of the worst located (furthest) node of a graph. The task is to find the optimal location of emergency centers so the demand of the worst located node would be served on time [5].

The supply function of center is characterized by term *depot*. The depot is for example the warehouse of material. Each node of a graph $G = (V, E, c, w)$ need $w(v)$ material units per time unit, while the unit costs for supplying the material are proportional to the travel distance. In this case, the location of centers is performed in such way, that the total transport costs of serving all nodes are minimized. Postal processing and distribution centers thus perform the *supply function*.

2. ALLOCATION MODELS

Probably the first location model dedicated to center location is in Goldman's manuscript [6] that extends the node optimality property of previous Hakimi's works [7, 8]. The two later researches about center location are from 1994 by Campbell [9] and O'Kelly and Miller [10]. These authors produced important advances in understanding hub systems and developed basic models focused primarily on minimizing flow cost and fixed facility costs, for rather constrained situations where a single cost discount is used for all flows between center nodes of the networks. The design of the hub level (highest level) network was determined by the location of the center nodes. Some researchers extended the basic models to include features such as direct origin-destination flows and capacities (Aykin) [11] additional objectives, including mode choice (O'Kelly and Lao) [12] and congestion (Gavish and O'Kelly) [13, 14].

With the burst of activity in location research through the 1990's two more recent review papers have appeared. Klinecicz [15] reviewed work in the telecommunications area that included the design of networks and the location of center nodes. Bryan and O'Kelly [16] surveyed work primarily in the context of air transportation and identified directions for future research.

The postal application has been discussed in two articles of Ernst and Krishnamoorthy [17, 18] based on the study of a metropolitan postal delivery system for Australia Post. Here the nodes represent post code districts, flow corresponds to mail volume, and the hub nodes are not only consolidation points but also used to sort mail.

A different hub location model for postal operations is described by Donaldson et al. [19]. They provide an integer programming model that allows routes via at most one hub, and use the model to evaluate the location of cross-docking sorting centers (hubs) for mail distribution in the USA. Their models include time constraints and allow direct origin-destination shipments.

To find the optimal location of depots in postal network it is suitable to use discrete network allocation models. One of the basic parameters for solving such problems is the very distance between nodes. From this point of view, it is possible to subdivide allocation models into two categories [20]:

- *Models based on maximum distance* (set covering, maximum covering, p-center).
- *Models based on total/average distance* (p-median, maximum, fixed charge location model).

For networks with supplying function it seems as the most appropriate to use the models based on total or average distance. These models are based on the average distance between depots and all demand nodes, ensuring that this average distance will be as minimal as possible (as well as the sum of travel distances to cover the whole network) [20]. Further in this article we will deal with p-median and uncapacitated fixed charge location model.

2.1. P-median location model

The aim of this model is to find the location of P facilities in network, serving all demands in a way ensuring the average transport cost to be minimal. The cost can be calculated by multiplying demands of certain vertex (network node) with distance between such node and the closets depot serving its demands. P-median location model works with following variables (Tab. 1) [7]:

Table 1

Input variables		Decision variables	
Notation	Description	Notation	Description
I	set of nodes with demands to be served	X_j	= 1 if we locate facility at candidate node j
J	set of candidate nodes for facility location		= 0 if not
h_i	demand at node i	Y_{ij}	= 1 if demands at node i are served by facility at node j
d_{ij}	distance between demand node i and candidate node j		= 0 if not
P	number of facilities to locate		

The p-median location problem can be formulated as follows:

$$\sum_i \sum_j h_i d_{ij} Y_{ij} \tag{1}$$

$$\sum_j Y_{ij} = 1 \quad \forall i \in I \tag{2}$$

$$\sum_j X_j = P \tag{3}$$

$$Y_{ij} \leq X_j \quad \forall i \in I, j \in J \tag{4}$$

$$X_j, Y_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \tag{5}$$

The optimizing function (1) minimizes the total demand weighted distance between individual nodes and closest centers. Constraint (2) requires each node i to be assigned to exactly one facility at node j . Constraint (3) ensures that exactly P facilities will be located. Constraint (4) states that demands at node i can only be assigned to a facility at location j if a facility is located at node j . Constraints (5) are standard integrality conditions [21].

2.2. Uncapacitated fixed charge location model

Fixed charge location model approach the issue of facility location based on minimizing the overall costs of implementation of selected variants. This model works with following assumptions [22]:

- Locating facility in candidate nodes might not imply the same fixed cost in each of them;
- Number of facilities to be located is not input of the model.

The solution of this problem will be the optimal number and position of facilities in a graph while minimizing the total costs of the model. When applying the uncapacitated fixed charge location model it is assumed that located facilities will have several times higher theoretical capacity than the necessary capacity in the serving area. Therefore it does not need to be included in calculations. This model works with following set of variables (Tab. 2) [22].

Table 2

Variables used in uncapacitated fixed charge location model

Input variables		Decision variables	
Notation	Description	Notation	Description
I	set of nodes with demands to be served	X_j	= 1 if we locate facility at candidate node j = 0 if not
J	set of candidate nodes for facility location		
h_i	demand at node i		
d_{ij}	distance between demand node i and candidate node j	Y_{ij}	fraction of demands at node i that is served by a facility at node j
f_j	fixed costs of locating candidate node j		
α	costs per distance unit per demand unit		

The uncapacitated fixed charge facility problem can be formulated as follows:

$$\sum_j f_j X_j + \alpha \sum_i \sum_j h_i d_{ij} Y_{ij} \quad (6)$$

$$\sum_j Y_{ij} = 1 \quad \forall i \in I \quad (7)$$

$$Y_{ij} \leq X_j \quad \forall i \in I; \forall j \in J \quad (8)$$

$$X_j \in \{0,1\} \quad Y_{ij} \geq 0 \quad \forall i \in I, j \in J \quad (9)$$

The optimizing function (6) minimizes the total costs, which is the sum of the fixed facility costs and transport costs (total demand weighted distance multiplied by costs per distance unit per demand unit). Constraint (7) requires each node i to be served. Constraint (8) ensures that demands at node i cannot be assigned to a facility at candidate node j unless we locate the facility at node j . Constraints (9) are the integrality and nonnegativity conditions [21].

2.3. Comparison of selected models

Based on above mentioned characteristics and parameters of both models, we can observe their similarities and differences. Both models are applicable on the same network, which is represented by complete weighted graph $G = (V, H, c, w)$ to simplify the calculations. Both have the same set of nodes and edges as well as evaluation (weight) of nodes and edges. Input variables include a set of nodes with demands to be served, set of candidate nodes for facility location, the demand value of individual nodes and the distance between each pair of nodes.

The input variable of p-median location model is also the number of facilities to be located on network. Algorithms solving this model are looking for mathematically optimal solution for given number of facilities and finish after finding it. This model does not count with the cost of building up the facilities; it tries to find the solution with minimal transport costs [21].

The uncapacitated fixed charge location problem does not have a specified number of facilities on input, which increases the variability of solution. On the other hand, it increases the computational complexity regarding the number of required iterations. The input variables include costs per distance unit per demand unit, which is relatively difficult to determine. Also the costs of building up facilities may differ in each node. This has to be also taken into account in the calculation. However, in addition to mathematically optimal solution, this model brings significant degree of economical optimality compared to the p-median location model. Such optimality is required when strategic decisions are made, similar to locating the postal processing and distribution centers. Therefore for the solution of decision problem addressed in this article we will use the uncapacitated fixed charge location model.

3. INPUT DATA AND UNDERLYING NETWORK

We are optimizing the existing postal network in the Slovak republic. There is a three-level structural variant of postal network implemented. The lowest level consists of regular post offices. These are connected only to the middle-level modes. The middle-level nodes perform function of mail concentrator and ensure a certain degree of mail processing. The postal items are then forwarded to high-level nodes, which are connected to each other. The process works similarly in the opposite direction – from higher to lower level nodes [23]. This process ensures the covering of the whole territory of Slovakia. It is obvious that this approach should be based on an analysis of the logistics of the local market [24].

The underlying infrastructure is represented by simplified model of a postal network, abstracted by a graph $G=(V, E, c, w)$. The set of vertices (nodes) V consists of all 41 existing middle-level nodes. The set of edges E represents road connections between nodes. Due to the strategic character and long-term impact of solution, we are taking into account completely built up network of planned highways and motorways while searching for road connection [25]. The labels of edges $c(e)$ have value of the shortest distance in kilometers. When determining the weight of nodes $w(v)$, we use the demographic and geographic characteristics of individual nodes and the covering region which they serve. We consider the following attributes [26, 27]:

- number of villages (or cities) in the middle-level region;
- number of villages (or cities) with postal offices in the middle-level region;
- number of citizens in the middle-level region;
- total area of the middle-level region;
- total distance between the middle-level region center and each villages (or cities) in the middle-level region;
- total distance between the middle-level region center and each villages (or cities) with postal office in the middle-level region;

The graph of addressed network placed on the map of covering regions is presented below (Fig. 1)



Fig. 1. Underlying graph representing the postal middle-level nodes and their road connections
Rys. 1. Wykres prezentujący pocztowe węzły średniego poziomu i ich połączenia drogowe

Above mentioned input data are applicable for almost any type of allocation model. The uncapacitated fixed charge location model uses another two characteristic variables, which are necessary to be set prior to obtaining the final solution [28].

One of those are *fixed costs* for the location and build of facility at certain node. We consider the standardized model of postal sorting center, which would have the same construction for all locations.

The basic value is set to 800 000 EUR, which resulted from similar projects implemented in practice in recent past. In addition, it is necessary to take into account the specifics of regions. We include the average price of building land and labor costs in individual regions in the calculation. Comparing to the national average values we obtain weights for each region. Multiplying the basic value by these weights will result in obtaining value of fixed costs for locating facility in each region.

Another required input variable is the *cost coefficient per distance unit per demand unit*. It is necessary to determine the weight of demand and transport costs. When calculating the demand weight, we used the available statistical data published by Universal Postal Union. The data on the average annual amount of mail per capita is important in this case. The obtained value 80.84 is divided by the number of days in a year to get the daily amount of mail – 0.2215. The transport costs per kilometer consist of fuel and oil consumption, vehicle wear and tear or depreciation. In addition we have to calculate with rent, driver salary, etc. [29]. Analyzing the available internet resources, we set the average value of transport costs to 0.22 EUR per kilometer for vehicle with a load capacity of 12 tones and one driver, which is generally used in transport of postal items between middle and high-level nodes. The final value of required coefficient is 0.04873 EUR per kilometer per demand unit.

4. OBTAINED RESULTS

The task of the model is to find the number and location of high-level nodes in the network while minimizing the costs of building the facilities and serving the demands of network. The solution can be achieved by using suitable heuristic algorithms. In this particular model there are the construction algorithms, which build on the general shape of the curve of total costs when obtaining solution. *ADD algorithm* approaches to achieving results based on principle that the total costs tend to decrease if appropriate facilities are added to the final set of optimized facilities location. *DROP algorithm* works on similar principle but with the difference that it approaches the curve of total cost from the opposite side. So the optimization process starts in a state where facilities are located in every node of the network. The improvement algorithm for this allocation problem is so-called exchange algorithm [30, 31]. To obtain the solution we use the DROP algorithm, since it can deliver better results than the ADD algorithm improved by the exchange algorithm.

To observe the optimization, it is essential to calculate the results for existing postal network on existing road infrastructure. The postal network of Slovak republic currently consists of four main processing and distribution centers representing high-level nodes location within the constructed graph (Fig. 1). Since the nature of used allocation model includes fixed costs for building facilities, these are not considered as we locate facilities at existing centers. The most important values resulting from such solution are transport costs and average distance to subordinate nodes. Obtained results are presented below (Tab. 3). Distribution of demands between individual facilities are far of equality as well as average distance, which results in high transport costs.

Table 3

Obtained solution for current postal network layout

Total results				
Number of facilities	Total demand weighted transport costs	Average distance	Average weighted distance	Covered demands
4	7 149 431.49 €	61.32 km	48.053 km	100%
Results for individual facilities				
Node	Demand weighted transport costs [€]	Average distance to demand nodes [km]	Number of assigned demands	Percentage of covered demands
1	2 350 340.49	66.91	1 069 100	37.09%
19	942 876.77	49.63	380 000	13.18%
24	1 380 062.84	52.40	638 800	22.16%
34	2 076 151.40	71.42	794 500	27.56%

Further solution considers completed network of highway roads. The fixed costs for building up new facilities are now included as we search for solution with lowest total costs of a model. Since the existing facilities are located entirely within urban areas of cities, these locations are not suitable for optimal operation as they subject to higher traffic flow and probability of adverse situations which may affect the reliability of postal item transport. Therefore the applied model calculate with fixed costs even for nodes with actually located facilities, but the costs incur for building up new facilities at suburban area. Obtained solution with calculated total costs is presented below (Tab. 4)

Table 4

Obtained solution for uncapacitated fixed charge location model

Final results of model					
Number of facilities	Total fixed costs	Total demand weighted transport costs	Total costs	Average weighted distance	Covered demands
3	2 740 200 €	6 609 581.26 €	9 349 781.26 €	47.057 km	100%
Results for allocated facilities					
Node	Fixed costs [€]	Demand weighted transport cost [€]	Average distance to demand nodes [km]	Number of assigned demands	Percentage of covered demands
3	940 400	2 791 171.559	55.46	1 163 700	40.37%
23	917 800	2 084 864.320	53.13	924 200	32.06%
35	882 000	1 733 545.385	52.67	794 500	27.57%

By application of uncapacitated fixed charge location model we found out that the minimum costs for given input values are achieved when locating three facilities at nodes representing the covering regions of cities Galanta, Martin and Prešov. The establishment of high-level nodes in these locations ensures the covering of all demands of the entire Slovak territory while minimizing the building and transport costs. The distribution of demands is more equal than current layout, as well as average distance to covered demand nodes. Resulting transport costs are considerably lower. The assignment of demand nodes to individual located facilities corresponding to the final solution are presented below (Fig. 2).

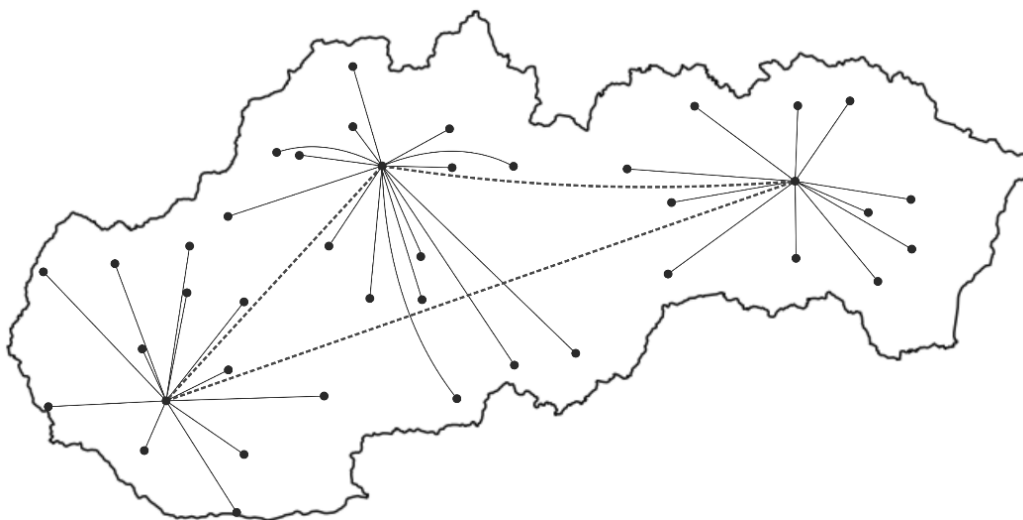


Fig. 2. High-level nodes with covered demand nodes for obtained solution
Rys. 2. Węzły wysokiego poziomu dla otrzymanego rozwiązania

5. CONCLUSION

Application of uncapacitated fixed charge facility location model on road infrastructure resulted in finding the location of three facilities while minimizing the costs of building them and costs of transporting postal items between nodes with demands. With the aid of available resources we were able to set the value of necessary input variables. Fixed costs of building a facility may not be the same in all nodes. This was taken into account in the calculation by coefficient considering the price of land at certain location etc. For used demand weight it was not possible to determine, whether it contains all types of postal items, whether they were included the postal items sent by legal entities, etc. It would be also essential to take into account the real volumes of postal items processed in individual nodes. Additional cost reduction would be possible by upgrading the existing high-level nodes instead of building new ones, but these are in a completely different location than results of used model. Considering the trend of increasing traffic, using facilities at current urban location can jeopardize the smooth flow of postal items in the future. Therefore it is recommended to build up new facilities in the suburban areas most preferably near highway exits, as already applied in Western Europe countries. The current layout of middle-level covering regions is obsolete as well, it would be necessary to deal with the issue of changing the covering area and reducing their number. After such optimization the presented model can bring even better results.

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