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VEHICLE CYCLE HIERARCHIZATION MODEL TO DETERMINE THE ORDER OF BATTERY ELECTRIC BUS DEPLOYMENT IN PUBLIC TRANSPORT

Summary. Battery electric buses (BEBs) are gaining ground in public transport, and the next decade is likely to witness a further increase in the numbers of these vehicles. The core problem of the BEB deployment process in public transport lies in how to deal with the technical limitations of this technology (primarily, a limited range of a bus). This paper outlines the model of vehicle cycle hierarchization dedicated to municipal transport planners and transit companies. The model is based on a multicriteria decision-making methodology that results in a hierarchy of vehicle cycles, ordered from the most to the least suitable to be operated by electric buses. The proof-of-principle demonstration of the model has been carried out in the Polish medium-sized city of Jaworzno.

1. INTRODUCTION

Transport problems in cities, the need to save energy, and the growing ecological awareness in society are forcing European and national authorities to adopt a policy of sustainable development [1]. Public transport is among the key policy areas to decrease the share of conventionally fueled vehicles. After many years of the dominance of diesel engines in urban buses, electric propulsion has been gaining ground since the second decade of the XXI century. Battery electric buses (BEBs) are starting to play a major role among low-carbon technologies in public transport. (There are also other types of alternatively fueled buses, like hydrogen ones, but they are not discussed in this paper.) The main operational drawback of BEBs is a technological inability to perform all vehicle cycles (the definition of which we will discuss later), given their state-of-the-art daily operation range without recharging [2÷4]. As BEBs are operated for several years, no common model of their operation is adopted by transit companies: various charging schemes are available [5÷9]. The lack of such a common operation model results from the variety of charging-related technological solutions available on the market. An attempt to deal with this issue is a subject of numerous research studies that we discuss in the next subsection.

1.1. Literature study

There is a multitude of research on BEBs in which various aspects of their operation are investigated. Due to the limitations in the length of this paper and the vastness of the subject matter, it is hardly possible to discuss all the above-mentioned issues in detail. We can distinguish the following research areas among them:

- Energy consumption calculation methods and research on battery performance.
- Battery charging and exchanging technologies.
- BEB deployment in cities and agglomerations.
- Ecological and economic assessment of the BEB operation.

Unlike for diesel buses, what affects the energy consumption of BEBs are route characteristics (including route gradient, the type of area through which a bus runs, and passenger loading) [10–12]. Diverse methodologies may be used for the calculation of the estimated energy consumption of the BEBs; these are presented, e.g., in [13–21]. As the energy consumption by BEBs affects the energy demand of cities and agglomerations, research is also conducted on a network-wide scale (e.g., in [22–23]).

There is a wide choice of battery charging technologies in the literature. Xylia and Silveira [24] gathered insights from the BEB demonstration projects, with a focus on charging-related issues (conductive and inductive charging technologies, slow- and fast-charging strategies, etc.). The research also included a survey for the stakeholders to specify the pros and cons of each solution in real-life conditions. One may find more research on this topic in [25–27]. Fewer studies involved battery exchanging (e.g., [28–29]), which is understandable, bearing in mind less extensive deployment of this technology.

In recent years, a growing interest has been observed in research related to BEB deployment [30–31]. The goal of these studies is to support the decision-making process in cities and agglomerations (e.g., to achieve emission targets when meeting budget constraints). Bezruchonak studied the geographical features of zero-emission mobility [32]. The research showed that the distribution of innovative technology and public support as well as clear strategic planning and interaction between transit companies and local authorities are of key importance in the process of BEB deployment. A few research papers included the transition process itself in various countries [33–36]. To support the BEB deployment, computer-aided tools were implemented. In [37], the BEB fleet transition problem was formulated as an integer linear program to gain insights into optimal transition plans. Islam and Lownes proposed a parallel bus replacement study in which a life cycle cost of owning and operating a BEB fleet (with infrastructure) is to be minimized [38]. A mixed-integer mathematical model was used by Uslu and Kaya to support decisions on location and capacity decision for BEB charging stations considering waiting times [39]. The Lagrangian Relaxation algorithm was adopted in [40] to optimize the battery electric bus charging facility location and to fit the BEB fleet size. In [41], deep learning methods were adopted when estimating BEB energy consumption on real-world data in the Polish municipality of Jaworzno. A genetic algorithm for the energy consumption minimization of BEBs was developed in [42], and a machine learning algorithm is applied in [43] for the same purpose. Wei et al. developed a spatial–temporal optimization model to minimize the BEB deployment cost while maintaining the existing bus schedule [44]. Topić et al. performed a simulation of conventional and BEB fleets over recorded driving cycles, which included charging management for transport electrification planning [45]. A methodology for bus route electrification based on a multi-objective optimization is presented in [46] and applied at the selected bus routes in the German city of Mannheim. In [47], the analytical hierarchy process method (AHP) and the technique for order preference by similarity to ideal solution (TOPSIS) were used for choosing the technical specifications of BEBs. The research on the BEB deployment is developed in the context of the bus fleet conversion process (to achieve as large a share of these vehicles as it is rational). An overview of the fleet conversion process and the basic decision-support algorithm for bus fleet conversion toward a 100% electric bus fleet is presented in [48]. This vehicle cycle-based approach is in line with this paper (especially the need for creating a vehicle cycle hierarchy ranked based on their ability to be operated by electric buses).

Other issues discussed in the literature are the environmental impact of the BEB operation (e.g., in [49–50]) and the economic assessment of the investment in BEBs (mainly in the form of a total cost of ownership calculation); see [51–54].

1.2. Problem description

It seems to be a struggle for many transit companies to deploy BEBs on the entire public transport network at once (mainly due to the limited budget). Hence, one may need to choose which transit lines to operate by BEBs first. It may be a preliminary step in a fleet conversion process, in which it is expected that along with the development of battery technology, the problem of insufficient coverage

will eventually be overcome [48]. As we want to tackle the technological limitations of BEB (the limited range, the need for charging, and better performance of these vehicles in heavy traffic) with the transit line characteristics, we rather need to look at this issue from the perspective of vehicle cycles. The latter are generally understood to mean the operation of a vehicle (bus) through the course of a day of transit service (including pull-out and pull-in), which consists of handling a sequence of bus stops (including depots and scheduled stopover times at terminus stops) under operation of one or more bus routes [2]. This definition implies an indirect selection of the transit lines to be operated by BEBs. Notwithstanding the aforesaid, by examining the issue from the vehicle cycle perspective, we directly link the technical limitations of BEBs with the route of the bus in the transit network.

The scientific problem may thus be defined briefly as follows: which vehicle cycles to choose as suitable to be BEB-operated, taking into account the limitations of this technology. To tackle this problem, a multi-criteria analysis of the vehicle cycle characteristics is carried out to hierarchize and thus prioritize them to be operated by BEBs in a selected public transport network.

2. EVALUATION OF VEHICLE CYCLE CHARACTERISTICS AFFECTING BATTERY ELECTRIC BUS DEPLOYMENT

The set of vehicle cycles is defined. The limitations imposed on conventional buses (drivers' working time regulations, required fuel level, etc.) are included in the vehicle cycle schedule. However, this does not apply to the specificity of the BEB operation. The purpose of the model is to search through this set to choose the ones suitable from the point of view of technological limitations of electric buses.

2.1. Modeling assumptions

In this paper, several assumptions of the public transport system operated by BEBs are made (Tab. 1.). These assumptions are necessary for modeling and because of the multiplicity of the public transport system design (in technical and organizational terms).

Table 1

Assumptions for the modeled BEB-operated public transport system

Serial No.	Wording of the modeling assumption
1.	A given bus schedule is not to be modified in the result of the BEB operation
2.	Passenger flows have been studied at the stage of bus schedule development and the appropriate bus capacity has been assigned
3.	The bus schedule is adjusted to the traffic conditions
4.	BEBs set off having a fully charged battery
5.	Energy consumption is described by the decrease of state of charge of the battery, set out in the scale $1 \div 100$ [%], assuming a linear decrease of the latter
6.	Two charging methods are included: plug-in battery charging in the depot and a pantograph-based opportunity charging across the transit network (at the selected stops, e.g., termini, at which power grid characteristics allow it)
7.	We involve neither battery exchange technology nor in-motion charging
8.	Infinite capacity of charging facilities is assumed (as a result, no queues at the charging facilities are modeled)
9.	Two vehicle cycles of the same route shall be treated separately if setting off at a different time
10.	We do not split the vehicle cycles – if possible, all routes are supposed to be operated by BEBs
11.	The reliability of the public transport system is not affected by the deployment of BEBs

The model assumes the absence of bus schedule modification resulting from the BEB operation. One may change the bus schedule to adjust it to the limitations of BEBs, but this may lead to an increase in the operational cost increase and may result in passenger dissatisfaction. We also simplify the energy consumption model to a linear one, to enable swift and efficient calculations for the entire transport system. One may also use another method of energy consumption, as indicated later in this paper. These methods may be of greater accuracy; however, performing calculations for all vehicle cycles in a bigger city may be cumbersome and time-consuming.

2.2. Selection of vehicle cycle characteristics

Based on the results of a survey conducted among Polish public transport companies as well as on the available literature, a dozen characteristics of vehicle cycles have been defined [10]. For the needs of the model, this number of characteristics has been limited according to the methodology briefly described below:

- Selection due to the determinants of the public transport system model operated by electric buses.
- Selection due to strong interrelationships with other characteristics.
- Selection due to the fact that the given group of characteristics is not affected by using electric propulsion.

For a detailed description of this methodology, see [10]. As a result of the elimination procedure, the following vehicle cycle characteristics have been selected:

1. Vertical profile of the vehicle cycle.
2. Number of bus stops at which battery charging is possible.
3. Length of time slots available for charging (at bus stops at which battery charging is possible).
4. Type of area through which the bus runs.

These characteristics will be treated as decision variables in the vehicle cycle hierarchization model.

2.3. Hierarchization model to determine the vehicle cycle electrification priority

The hierarchization model (*MHL*) of vehicle cycles to rank them based on their ability to be operated by BEBs can be written as a 4-tuple of a model of the public transport network (*MSTP*), bus schedule (*RJ*), battery state model (*MSB*), and vehicle cycle hierarchy (*HL*).

$$MHL = (MSTP, RJ, MSB, HL). \quad (1)$$

MSTP covers a directed graph G of the structure of a public transport network and functions F_G specified on both arcs and vertices of this graph; $MSTP = (G, F_G)$. The graph G is an ordered pair $(V(G), A(G))$, where $V(G)$ stands for a set of vertices of graph G , while $A(G)$ represents a set of its arcs. Vertices stand for bus stops $V(G) = v_{\alpha_1}, v_{\alpha_2}, \dots, v_d, \dots, v_{\alpha_n}$ and arcs for inter-stop sections $A(G) = \{a_{\beta_1}, a_{\beta_2}, \dots, a_t, \dots, a_{\beta_m}\}$. (The notation $v_{\alpha_1}, \dots, v_{\alpha_n}$ instead of v_1, \dots, v_n – and for arcs accordingly – indicates that neither vertices nor arcs need to be numbered sequentially.) The cardinality of the set $V(G)$ is denoted $v(G)$, and cognately $a(G)$ stands for the number of arcs of $A(G)$. There are n bus stops and m inter-stop sections; $v(G) = n$ and $a(G) = m$. Further, function ψ_G defines an arc's two endpoints $\psi_G(a_t) = (v_k, v_l)$, $t \in \{\beta_1, \dots, \beta_m\}$, $k, l \in \{\alpha_1, \dots, \alpha_n\}$, where v_k stands for the initial point of the arc a_t and v_l is the end of this arc. The latter is interpreted in a way that function ψ_G assigns ordered pairs of vertices (bus stops) to the arcs (inter-stop sections), provided that a bus link between these two vertices exists. To simplify the notation, arc a_t , where $\psi_G(a_t) = (v_k, v_l)$, $t \in \{\beta_1, \dots, \beta_m\}$, $k, l \in \{\alpha_1, \dots, \alpha_n\}$, is denoted $v_k v_l$ later in this model.

According to the standard graph theory [55], a path from v_0 to v_n is defined as a finite sequence of arcs $v_0 v_1, v_1 v_2, \dots, v_{n-1} v_n$, which is denoted $v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_{n-1} \rightarrow v_n$. A nature of the bus service in the transport network (e.g., transport operations on recurring bus lines, including cycles) leads us to the formal definition of the vehicle cycle as the finite sequence of arcs of graph G . With this definition, the occurrence cycles (in the sense of the graph theory) are permitted. It is also permissible that the first and last bus stop is the same vertex of the graph G . Moreover, multiple occurrences of the same arcs and vertices are allowed.

A map $f_W: N \rightarrow R^5$ is defined as follows:

$$f_W(v_d) := (\lambda(v_d), \phi(v_d), h(v_d), kspw(v_d), wtp(v_d)) \equiv (\lambda_d, \phi_d, h_d, kspw_d, wtp_d), \quad (2)$$

where $d \in \{\alpha_1, \dots, \alpha_n\}$, λ_d is a longitude of the d -th vertex, ϕ_d is a latitude of the d -th vertex, h_d is an altitude of the d -th vertex, $kspw_d$ is a traffic speed zone in which the d -th vertex is situated, and wtp_d is a technical equipment of the d -th vertex with a value of 1 if there is a pantograph-based opportunity charging facility at the bus stop, 2 if there is a plug-in charging facility at the bus stop (or in the depot), and 0 otherwise.

For all the vertices of the graph, we define the following function:

$$F_W \begin{bmatrix} v_{\alpha_1} \\ \vdots \\ v_d \\ \vdots \\ v_{\alpha_n} \end{bmatrix} := \begin{bmatrix} (\lambda_{\alpha_1}, \phi_{\alpha_1}, h_{\alpha_1}, kspw_{\alpha_1}, wtp_{\alpha_1}) \\ \vdots \\ \lambda_d, \phi_d, h_d, kspw_d, wtp_d \\ \vdots \\ \lambda_{\alpha_n}, \phi_{\alpha_n}, h_{\alpha_n}, kspw_{\alpha_n}, wtp_{\alpha_n} \end{bmatrix}. \quad (3)$$

According to the function $\psi_G(a_t)$, every arc of graph G may be interpreted as the ordered pair of arcs (bus stops) and be denoted $(v_k v_l)$ for $k, l \in \{\alpha_1, \dots, \alpha_n\}$. By $A \in R_n^n$, we denote a matrix of all the arcs of graph G

$$A = \begin{bmatrix} (v_1 v_1) & \cdots & (v_1 v_n) \\ \vdots & \ddots & \vdots \\ (v_n v_1) & \cdots & (v_n v_n) \end{bmatrix}. \quad (4)$$

A map $f_L: N^2 \rightarrow R^3$ is defined as follows:

$$f_L(v_k v_l) := (len_{kl}, \Delta h_{kl}, kspl_{kl}), \quad (5)$$

where $k, l \in \{1, \dots, n\}$, len_{kl} is a length of the arc $v_k v_l$, Δh_{kl} is a sum of the uphill lengths of the arc $v_k v_l$, and $kspl_{kl}$ is a traffic speed zone in which the arc $v_k v_l$ is situated. This means that for all the arcs of the graph G , we define the following function:

$$F_L(A) = \begin{bmatrix} 0 & (len_{12}, \Delta h_{12}, kspl_{12}) & \cdots & (len_{1n}, \Delta h_{1n}, kspl_{1n}) \\ (len_{21}, \Delta h_{21}, kspl_{21}) & 0 & \cdots & (len_{2n}, \Delta h_{2n}, kspl_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (len_{n1}, \Delta h_{n1}, kspl_{n1}) & (len_{n2}, \Delta h_{n2}, kspl_{n2}) & \cdots & 0 \end{bmatrix}. \quad (6)$$

The next element of the *MSTP* – bus schedule (denoted: *RJ*) – covers a set of vehicle cycles by which buses operate at a given time. The vehicle cycle can also be identified by a finite sequence of vertices (bus stops) through which the bus runs. The bus schedule *RJ* includes a set of all the vehicle cycles (*LIN*) and a function $T(LIN)$ that assigns a given departure time to the vehicle cycle, that is

$$RJ = (LIN, T(LIN)), \quad (7)$$

where *LIN* is a set of all the $c \in N$ vehicle cycles, namely,

$$LIN = \{lin_b: b = 1, \dots, c\}, \quad (8)$$

where lin_i is a sequence $v_{1(lin_b)}, \dots, v_{p(lin_b)}$ for $p(lin_b) \in N$ and $p(lin_b)$ is identified with the number of bus stops through which a bus runs when performing a vehicle cycle. As the function $T(LIN)$ assigns a time of departure to the vehicle cycle, we may easily calculate the departure times from subsequent bus stops on determining the inter-stop time values.

The battery state model *MSB* describes the state of charge (SOC) of the battery. The theoretical operational bus range is denoted by z . However, to avoid battery discharging, one should assume a reserve of (at least) 10% of the capacity for further calculations. We assume linear energy consumption at a rate of $\eta\%$ per kilometer. Such an approach is a vast simplification applied to facilitate the model to the end-users in transit companies and local transport authorities. Nevertheless, the selected factors that may influence the energy consumption on routes are included in the evaluation of the vehicle cycle characteristics (namely, the vertical profile of the vehicle cycle and the type of area through which the bus runs). To make the energy consumption calculations more accurate, one may take advantage of other energy consumption models, e.g., the one in [56], where a LabVIEW-based model is developed to estimate the SOC.

We assume that the battery is charged at the already defined bus stops equipped with charging infrastructure; $wtp_d \neq 0$. Battery charging time is denoted by t_{lad} and is given by

$$t_{lad} = \begin{cases} \zeta_1 \%/min, & \text{if } wtp_d = 1 \\ \zeta_2 \%/min, & \text{if } wtp_d = 2 \end{cases} \quad (9)$$

where ζ_1, ζ_2 are the recharging speeds for pantograph and plug-in charging, respectively. We also introduce a variable t_{min} , meaning the minimum charging time necessary to charge the battery up (not to model the time slots too short to perform charging). A total operational range of a bus performing a vehicle cycle may – and often should – be of a greater value than z if only:

- Within the path of the i -th vehicle cycle, there will be at least two bus stops for which $wtp_d \neq 0$ for every z kilometer.
- Bus schedule of the i -th vehicle cycle will secure a stopover for charging, having the condition $t_{lad} > t_{min}$ fulfilled.

A bus will be able to serve a given vehicle cycle only if the battery level during the charging stopover increases to such a level as to allow the vehicle to proceed to the subsequent bus stop for which $wtp_d \neq 0$ and $t_{lad} > t_{min}$. Another situation in which a bus can perform a vehicle cycle is when the range of the bus z is of a greater value than the total length of the vehicle cycle.

To determine the hierarchy of vehicle cycles liable to be operated by electric buses HL , the following four constraints have been defined:

1. Irrespective of weather conditions, the BEB range is set as z kilometers; $\sum_{v_k v_l \in \rho_b} len_{kl} \leq z$.
2. The state of charge of the battery (SOC) must not be lower than 10%; $\theta \geq 10\%$.
3. For the intended BEB range, there must be a minimum of two charging facilities for every z km of the route of the vehicle cycle; $\forall (\sum_{v_k v_l \in \rho_b} len_{kl} = z) |\{t_d \in Y_b: wtp(v_d) \neq 0, t_{lad} > t_{min}\}| \geq 2$.
4. Each charging slot needs to last longer than or equal to the minimum charging time; $\forall t_{lad} [t_{lad} \geq t_{min}]$.

The set of functions of vehicle cycles that fulfills the above-mentioned criteria is denoted by LIN^*

$$LIN^* = \{lin_i: i = 1, \dots, s\}, \quad (10)$$

where lin_i is a sequence of $v_1(lin_i), \dots, v_p(lin_i)$ for $p(lin_i) \in N$, while $p(lin_i)$ is a number of vertices of the s -th vehicle cycle. Note that the latter is a subset of LIN .

The next step is to define a hierarchization procedure that will result in a hierarchy of vehicle cycles HL . Let us define the following four decision variables on the grounds of the variable selection procedure:

$$X_1(LIN^*) = \begin{bmatrix} x_1(lin_1) \\ \vdots \\ x_1(lin_i) \\ \vdots \\ x_1(lin_s) \end{bmatrix}, \quad \dots, \quad X_4(LIN^*) = \begin{bmatrix} x_4(lin_1) \\ \vdots \\ x_4(lin_i) \\ \vdots \\ x_4(lin_s) \end{bmatrix}, \quad (11)$$

where: $X_1(LIN^*)$ is the vertical profile of the vehicle cycle; $X_2(LIN^*)$ is the number of bus stops at which battery charging is possible (called ‘number of technical stops’); $X_3(LIN^*)$ is the length of time slots available for charging (called ‘charging indicator of the vehicle cycle’), and $X_4(LIN^*)$ is the type of area through which the bus runs.

We define the vertical profile of the vehicle cycle (12) as a ratio of the length of the vehicle cycle, reduced by the sum of the uphill lengths in the numerator

$$x_1(lin_i) = \frac{\sum_{v_k v_l \in \rho_i} len_{kl} - \sum_{v_k v_l \in \rho_i} \Delta h_{kl}}{\sum_{v_k v_l \in \rho_i} len_{kl}} \in [0,1]. \quad (12)$$

The number of technical stops (13) indicates the number of bus stops at which battery charging is possible (technical stops). This decision variable is interpreted as the number of stops in the course of the vehicle cycle for which a charging facility exists, and the minimum charging time condition is fulfilled

$$x_2(lin_i) = |\{t_d \in Y_i: wtp_d \neq 0, t_{lad} > t_{min}\}| \in \{0, \dots, p(lin_i) - 2\}, \quad (13)$$

where Y_i is a set of all bus stops of the i -th vehicle cycle and $p(lin_i) - 2$ is the number of bus stops in the course of the vehicle cycle (reduced by the first and last one – the depot).

The charging indicator of the vehicle cycle (14) is interpreted as the total time of time slots at the bus stops equipped with charging facilities. It is a weighted sum of potentially available time slots at bus stops for which $wtp_d \neq 0$

$$x_3(lin_i) = \frac{1}{2} \sum \{t(v_d): [t(v_d) \geq t_{min} \wedge wtp(v_d)=2]\} t(v_d) + \sum \{t(v_d): [t(v_d) \geq t_{min} \wedge wtp(v_d)=1]\} t(v_d). \quad (14)$$

The type of area through which the bus runs is a value of the function that determines the following:

- The affiliation of individual bus stops within the i -th vehicle cycle to one of three traffic speed zones denoted $kspl_r(v_k v_l)$:
 - $kspl_1(v_k v_l)$ – built-up areas in city centers with the maximum allowed speed limited to 50 km/h.
 - $kspl_2(v_k v_l)$ – built-up areas outside city centers with the maximum allowed speed limited to 50 km/h.
 - $kspl_3(v_k v_l)$ – roads of increased speed limit (above 50 km/h).

(The concept of traffic speed zones is elaborated in [57].)

- The number of scheduled and unscheduled stops of the vehicle. The scheduled stops are identified with the number of bus stops to be handled under the vehicle cycle, including request stops. To include the lack of obligation to stop at request stops in the model, an indicator w_{NZ_r} is introduced. The value of the latter depends on the traffic speed zone type, to which the arc is affiliated, and the time of operation. The unscheduled stops may result from various reasons e.g., signalized intersection, pedestrian crossings, etc. These kinds of stops are included by an indicator w_{Zat_r} . Like w_{NZ_r} , the value of w_{Zat_r} depends on the traffic speed zone type, to which the arc is affiliated, and the time of operation.

The value of the type of area through which the bus runs (15) is identified with the value of the theoretical number of stops of the bus per length unit of the i -th vehicle cycle

$$x_4(lin_i) = \frac{1 + \sum_{r=1}^3 [\sum_{(v_k v_l) \in \Omega_r} (w_{NZ_r}(v_k v_l) + w_{Zat_r}(v_k v_l))]}{\sum_{v_k v_l \in \rho_i} len_{kl}}, \quad (15)$$

where Ω_r is a set of all bus stops affiliated to the r -th $kspl_r(v_k v_l)$.

To ensure comparability of the decision variables, normalization is necessary. It consists of dividing the value of a decision variable by its maximum value. The normalized values will be denoted $X^*_1(LIN^*), X^*_2(LIN^*), X^*_3(LIN^*), X^*_4(LIN^*)$

$$X^*_1(LIN^*) = \begin{bmatrix} x_1(lin_1) \\ \vdots \\ x_1(lin_i) \\ \vdots \\ x_1(lin_s) \end{bmatrix}, \dots, X^*_4(LIN^*) = \begin{bmatrix} x_4(lin_1) \\ \vdots \\ x_4(lin_i) \\ \vdots \\ x_4(lin_s) \end{bmatrix}. \quad (16)$$

It is beneficial for the decision variable $X^*_1(LIN^*)$ to be maximized as flat routes are preferred instead of hilly ones. It is also propitious to have as many bus stops equipped with charging facilities as possible. This means that we should strive to maximize the decision variable $X^*_2(LIN^*)$. Such a construction of the model enables the vehicle cycles to have the lowest possible energy consumption (resulting from the flatness of the route), on the one hand, and leads to the possibility of frequent service of technical stops on the other. Also, it is advantageous for the decision variable $X^*_3(LIN^*)$ to be maximized due to the need to ensure a seamless vehicle cycle operation. Finally, given the environmental benefits associated with avoiding the local emissions as a result of a BEB operation (instead of a diesel bus), the value of the type of area through which the bus runs $X^*_4(LIN^*)$ is to be minimized as well. The latter also results from the technical characteristics of the electric engine, which works well under heavy traffic conditions (numerous decelerations, stoppings, and accelerations) that take place in the central areas of cities and agglomerations.

There are four decision criteria K_1, K_2, K_3, K_4 established in the model. These are dependent on the subsequent decision variables $X^*_1(LIN^*), X^*_2(LIN^*), X^*_3(LIN^*), X^*_4(LIN^*)$. The decision criteria are assigned weights $\delta_1, \delta_2, \delta_3, \delta_4$ (whose values are to be assigned using analytical methods, foresight analysis, or based on further research for a specific spot). Hence, the objective function takes the following form:

$$F(LIN) = \delta_1 \cdot X^*_1(LIN^*) + \dots + \delta_4 \cdot X^*_4(LIN^*), \quad (17)$$

which can also be written as

$$F \begin{bmatrix} lin_1 \\ \vdots \\ lin_i \\ \vdots \\ lin_s \end{bmatrix} = \delta_1 \cdot \begin{bmatrix} x^*_1(lin_1) \\ \vdots \\ x^*_1(lin_i) \\ \vdots \\ x^*_1(lin_s) \end{bmatrix} + \dots + \delta_4 \cdot \begin{bmatrix} x^*_4(lin_1) \\ \vdots \\ x^*_4(lin_i) \\ \vdots \\ x^*_4(lin_s) \end{bmatrix}. \quad (18)$$

While meeting constraints 1–4, a hierarchization process is carried out, by ordering the value of the criterion function set out in formula (18) in nondecreasing order.

Note that in the *MHL*, no direct economic evaluation is performed. However, this does not mean that the economic impact of the planned investment cannot be assessed. Given the prominence of the economic aspect in the entire BEB deployment process, let us briefly discuss this issue.

The key parameter of the cost-related calculations of the BEB deployment is the total cost of ownership (TCO). It may be defined, e.g., as the total cost paid by consumers (in our case, primarily by the transit companies) from acquisition to disposal [58]. As already set out in the literature study, there are numerous methodologies to assess TCO values. Due to the specificity of this model – see Table 1 – one may use the *TCOModel* of the PLATON Toolkit [59]. This tool is content-wise closely related to the vehicle cycle hierarchization method presented in this paper. Because of the space constraints in this paper, no detailed description of the model can be outlined. The highlights of the TCO model discussed are as follows [59]:

- The model comprises both the static calculation (S-TCO) for the one-off conversion process and the dynamic one (D-TCO) whose purpose is to analyze the subsequent batches of BEBs over time.
- Both these models include technical, economic, financial, operation, and maintenance input variables (note that some of these are the outputs of the *MHL* model).
- The outputs of the *TCOModel* comprise the present values of costs related to the BEB fleet, infrastructure, operating, and external costs as well as the liquidation value.

The interdependencies between the subsequent elements of the model itself (*MHL*) and the *TCOModel* are depicted in Fig. 1. The economic assessment of the deployment solution concerns iterative repetitions of the *MHL* outputs using the *TCOModel*, as long as the economic influence of the investment is acceptable for the investor.

3. PROOF-OF-PRINCIPLE REAL-LIFE IMPLEMENTATION OF THE MODEL

To implement the vehicle cycle hierarchization model, a transit network operated by the Polish transit company PKM Jaworzno has been selected. To implement the model, the following activities were carried out:

- Modeling the public transport network using graph theory based on the scheme of bus routes of PKM Jaworzno [60] and the OpenStreetMap® data [61].
- Calculations to obtain a vertical profile of subsequent inter-stop sections to determine the decision variable $X_1(LIN^*)$ using OpenStreetMap® [61] and data obtained from PKM Jaworzno.
- Bus (vehicle cycle) schedule analysis to establish the total number of technical stops and the total time of time slots at the bus stops equipped with charging facilities to determine the decision variables $X_2(LIN^*)$ and $X_3(LIN^*)$, keeping in mind the following technical equipment:
 - Plug-in charging facilities ($wtp_d = 2$) are only located in the depot.
 - Pantograph charging stations ($wtp_d = 1$) are deployed in 7 termini scattered around Jaworzno (at the following bus stops: Krakowska Pętla, Szpital Pętla, Podłęże Osiedle, Łubowiec Pętla, Osiedle Stałe, Szczakowa Dworzec PKP, Buczyna Astrów).
- Delimitation of the area based to establish three traffic speed zones – see Eq. (15) – and empirical studies to determine the values of w_{Nz_r} and w_{Zat_r} to obtain $X_4(LIN^*)$ for all these zones (the results of these studies are presented in [57]).

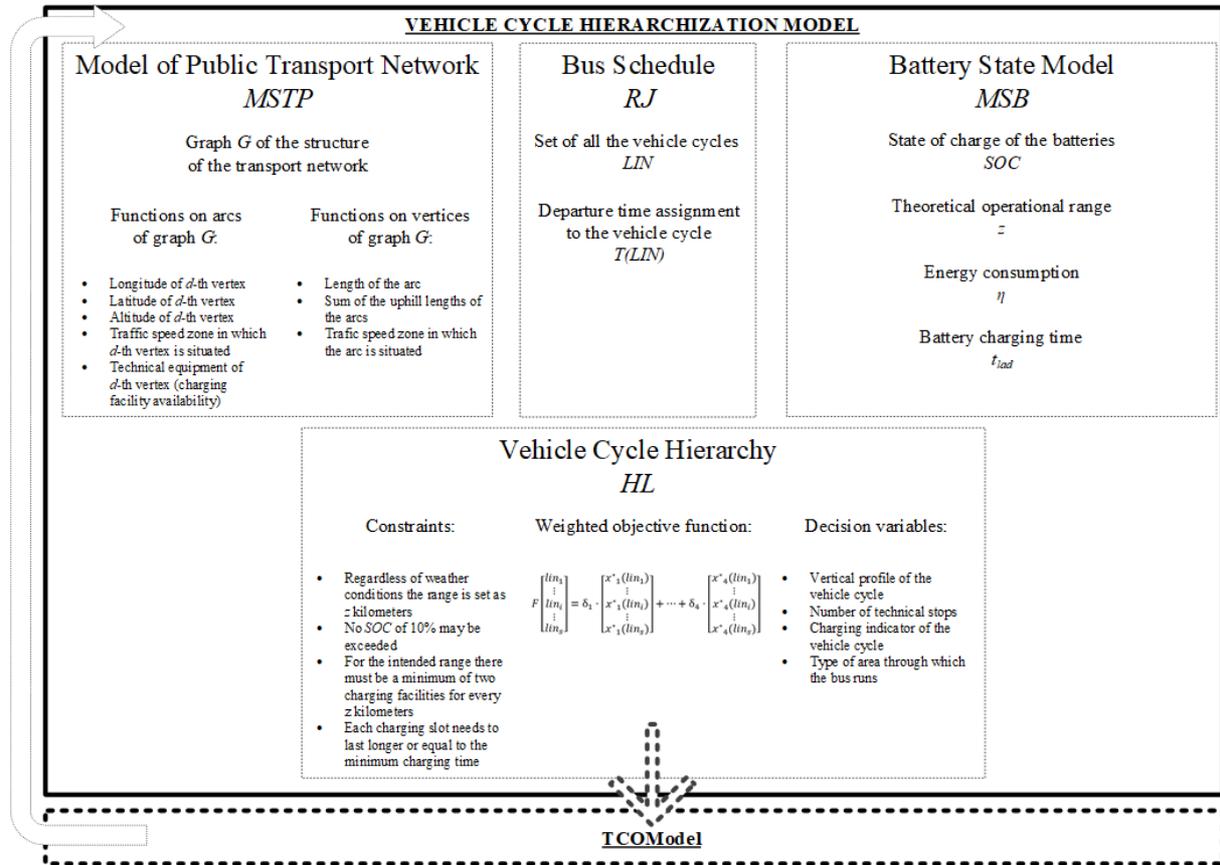


Fig. 1. Relationships between the elements of the vehicle cycle hierarchization model and the *TCOModel*

Based on the real-life BEB operating experience in Jaworzno, the variables take the following values. The theoretical operational range of electric buses z is equal to 100 km to make the vehicle cycle operable even in adverse weather conditions during winter operations (note that the value of z is significantly lower than the catalogue data). The energy consumption η is equal to 1%/km per a 12-meter bus, and the battery charging time t_{lad} is 1%/min if $wtp_d = 1$ and 0.5%/min if $wtp_d = 2$.

This transit company operated 57 vehicle cycles at the time of analysis, $LIN = \{lin_b: b = 1, \dots, 57\}$. For modeling purposes, 452 vertices (whose numbering is not sequential to keep their official ID numbers) and 611 arcs of the graph G were defined. Next, the functions were assigned to them according to Eqs. (3) and (6):

$$F_W \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ \vdots \\ v_{992} \end{bmatrix} = \begin{bmatrix} (50.191355, 19.285887, 284, 2, 2) \\ (50.192677, 19.284854, 283, 2, 1) \\ (50.184290, 19.286066, 279, 2, 0) \\ \vdots \\ (50.215782, 19.255243, 286, 2, 0) \end{bmatrix}, \quad (19)$$

$$F_L(A) = \begin{bmatrix} 0 & (10108, 50, 2) & \dots & (278, 0, 2) & (10108, 2050, 2) \\ (299, 50, 2) & 0 & \dots & (888, 200, 2) & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (278, 50, 2) & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}. \quad (20)$$

Based on the analysis carried out, only 12 out of 57 vehicle cycles met the constraints of the model ($z = 100$ [km], $t_{min} = 15$ [min]); $LIN^* = \{lin_i: i = 1, \dots, 12\}$. In the next steps, the decision variables $X^*_1(LIN^*)$, $X^*_2(LIN^*)$, $X^*_3(LIN^*)$, $X^*_4(LIN^*)$ have been calculated and then normalized; see formula (16). For the implementation of the model, it was also necessary to determine the weight

values. Based on the questionnaire sent across Polish public transport operators, the following weights have been determined: $\delta_1 = 0.19$, $\delta_2 = 0.23$, $\delta_3 = 0.29$, and $\delta_4 = 0.29$. Hence, the objective function takes the following form:

$$F(LIN) = 0.19 \cdot X^*_1(LIN^*) + 0.23 \cdot X^*_2(LIN^*) + 0.29 \cdot X^*_3(LIN^*) + 0.29 \cdot X^*_4(LIN^*). \quad (21)$$

The calculated values of the above equation are as follows:

$$F \begin{bmatrix} lin_8 \\ lin_4 \\ lin_{10} \\ \dots \\ lin_{11} \end{bmatrix} = \begin{bmatrix} 0.831 \\ 0.817 \\ 0.794 \\ \dots \\ 0.570 \end{bmatrix}. \quad (22)$$

Based on the above analysis, it was concluded that the vehicle cycles denoted by lin_4 , lin_8 , and lin_{10} are the most suitable to be operated by BEB, while lin_{11} is the least susceptible to be operated by this type of propulsion.

Verification of the model was conducted by comparing the results of the implemented model (vehicle cycle hierarchy) with the outcome of the allocation of electric buses to vehicle cycles by PKM Jaworzno. This verification showed the correctness of the developed model.

4. DISCUSSION AND CONCLUSIONS

The developed model represents the first scientific approach to study the BEB deployment process from the perspective of vehicle cycle hierarchization. The model works for a public transport system with defined assumptions (e.g., no bus schedule is modified, plug-in and pantograph charging is available, no split of vehicle cycles) and is based on four decision variables, chosen from the multiplicity of vehicle cycle characteristics.

The model itself is written as a 4-tuple of a model of the public transport network (*MSTP*), bus schedule (*RJ*), battery state model (*MSB*), and vehicle cycle hierarchy (*HL*). This model should not be used as the only BEB deployment tool. Rather, its use will be useful for the vehicle cycle evaluation in the limited budget case. (If there had been enough money for the electrification of the entire transit network, including as many charging facilities as required, no vehicle cycle analysis would have been needed.) When creating the model, the possibility of its operation in dense and extensive transport networks was considered; therefore, the model used to calculate energy consumption is not as accurate as in other studies. It is planned to apply more detailed methods of calculating energy consumption. Here, however, we need ensure as to not to extend the calculation time significantly. This will make the model much less useful for real-life applications, especially in larger cities and agglomerations.

The model presented in this paper should not, however, be considered as a standalone the only tool in the BEB deployment process. It is only used to support the assignment of BEBs to vehicle cycles in case of a limited budget (until battery technology is evolved enough to simply convert conventional vehicles into electric ones, without considering charging-related issues). It needs to be noticed here that in the BEB deployment schemes, buses are seldom exchanged in a single batch. Rather, based on available funds, technological limitations, and the current state of battery technology, subsequent batches of buses are purchased. The determination of the size of these batches may also be the result of the presented procedure. In this process, the *TCOModel* may be useful to assess the economic impact of the investment. The iterative use of the vehicle cycle hierarchization procedure may turn out to be helpful in the bus fleet transition process into a fully electric bus fleet (or any other agreed share of BEBs).

The advantage of this vehicle cycle hierarchization procedure is its versatility – there are no limitations for the model implementation in public transport systems, except for the data preparation (which may be cumbersome and time-consuming). However, the presented model is sensitive to the selection of weights of the criterion function. These weights may be determined by public transport operators, based on the knowledge of the specificity of a given public transport network. Further studies

are needed to establish these values for a general case (if the latter is possible at all, due to the diversity in public transport network design).

The proof-of-principle demonstration indicated the possibility of using the hierarchization procedure in practice. The calculated results are in line with the professional experience of PKM Jaworzno's employees and their calculations. As the number of BEBs operated in Jaworzno will increase soon, the hierarchization procedure may be reinitiated soon to indicate subsequent vehicle cycles suitable for electrification. Furthermore, everything indicates that the model is ready to be implemented in other public transport networks, including more complex and extensive ones.

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